

### Trigonometric Identities

#### Reciprocal Trigonometric Identities

$$\sin \theta = 1/\csc \theta \text{ or } \csc \theta = 1/\sin \theta$$

$$\cos \theta = 1/\sec \theta \text{ or } \sec \theta = 1/\cos \theta$$

$$\tan \theta = 1/\cot \theta \text{ or } \cot \theta = 1/\tan \theta$$

#### Pythagorean Trigonometric Identities

$$\sin^2 a + \cos^2 a = 1$$

$$1 + \tan^2 a = \sec^2 a$$

$$\operatorname{cosec}^2 a = 1 + \cot^2 a$$

#### Ratio Trigonometric Identities

$$\tan \theta = \sin \theta / \cos \theta$$

$$\cot \theta = \cos \theta / \sin \theta$$

#### Sum and Difference of Angles Trigonometric Identities

$$\sin(\alpha + \beta) = \sin(\alpha) \cdot \cos(\beta) + \cos(\alpha) \cdot \sin(\beta)$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

### Derivation Formula

#### Product Rule

$$(d/dx)(fg) = fg' + gf'$$

#### Quotient Rule

$$(d/dx)(f/g) = (gf' - fg')/g^2$$

#### Chain Rule

$$y = f(g(x)), \text{ then } y' = f'(g(x)) \cdot g'(x)$$

### Derivatives of Trigonometric Functions

$$\text{If } f(x) = \sin x, \text{ then } f'(x) = \cos x$$

$$\text{If } f(x) = \cos x, \text{ then } f'(x) = -\sin x$$

$$\text{If } f(x) = \tan x, \text{ then } f'(x) = \sec^2 x$$

$$\text{If } f(x) = \cot x, \text{ then } f'(x) = -\operatorname{csc}^2 x$$

$$\text{If } f(x) = \sec x, \text{ then } f'(x) = \sec x \tan x$$

$$\text{If } f(x) = \operatorname{csc} x, \text{ then } f'(x) = -\operatorname{csc} x \cot x$$

#### Example 1

$$\begin{aligned} f(x) &= \tan x \\ &= \frac{\sin x}{\cos x} \\ f'(x) &= \frac{\cos x \cdot \cos x - (-\sin x) \sin x}{\cos^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \end{aligned}$$

#### Example 2

$$\begin{aligned} y' &= 3x^2 \cot x + x^3 (-\operatorname{csc}^2 x) \\ &= 3x^2 \cot x - x^3 \operatorname{csc}^2 x \end{aligned}$$

#### Examples

$$g(x) = 3 \sec(x) - 10 \cot(x)$$

#### Derivative of sin x

$$\begin{aligned} \frac{d(\sin x)}{dx} &= \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{(x+h) - x} \\ &= \lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} \sin x + \frac{\sin h}{h} \cos x \\ &= (0) \sin x + (1) \cos x \\ &= \cos x \end{aligned}$$

#### Derivative of cos x

### Derivative of sec x (cont)

$$\sec x = 1/\cos x$$

$$\tan x = \sin x / \cos x$$

$$(\cos x)' = -\sin x$$

$$(\sec x)' = (1/\cos x)' = (-1/\cos^2 x)$$

$$(\cos x)'$$

$$= (-1/\cos^2 x) \cdot (-\sin x)$$

$$= \sin x / \cos^2 x$$

$$= (\sin x / \cos x) \cdot (1/\cos x)$$

$$= \tan x \sec x$$

$$\text{Therefore, } d(\sec x)/dx = \tan x \sec x$$

$$\sec x$$

### Derivative of cot x

We will determine the derivative of cot x using the quotient rule.

We will use the following formulas and identities to calculate the derivative:

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$\cot x = \cos x / \sin x$$

$$\cos 2x + \sin 2x = 1$$

$$\operatorname{cosec} x = 1/\sin x$$

$$(\cot x)' = (\cos x / \sin x)'$$

$$= [(\cos x)' \sin x - (\sin x)' \cos x] / \sin^2 x$$

$$= [-\sin x \cdot \sin x - \cos x \cdot \cos x] / \sin^2 x$$

$$= (-\sin^2 x - \cos^2 x) / \sin^2 x$$

$$= -1/\sin^2 x$$

$$= -\operatorname{cosec}^2 x$$

$$\text{Therefore, } d(\cot x)/dx = -\operatorname{cosec}^2 x$$

$$\operatorname{cosec}^2 x$$

### Derivative of cosec x

We will determine the derivative of cosec x using the chain rule.

We will use the following formulas and identities to calculate the derivative:

$$\operatorname{cosec} x = 1/\sin x$$

$$\cot x = \cos x / \sin x$$

$$(\sin x)' = \cos x$$

$$(\operatorname{cosec} x)' = (1/\sin x)' = (-1/\sin^2 x)$$

$$= (-1/\sin^2 x) \cdot (\cos x)$$

$$= -\cos x / \sin^2 x$$

$$= -(\cos x / \sin x) \cdot (1/\sin x)$$

$$= -\cot x \operatorname{cosec} x$$

$$\text{Therefore, } d(\operatorname{cosec} x)/dx = -\cot x \operatorname{cosec} x$$

$$\cot x \operatorname{cosec} x$$

$$\begin{aligned}
\frac{d(\cos x)}{dx} &= \lim_{h \rightarrow 0} \frac{\cos(x+h) - \cos x}{(x+h) - x} \\
&= \lim_{h \rightarrow 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h} \\
&= \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} \cos x - \frac{\sin h}{h} \sin x \\
&= (0) \cos x - (1) \sin x \\
&= -\sin x
\end{aligned}$$

### Derivative of sec x

We will determine the derivative of sec x using the chain rule. We will use the following formulas and identities to calculate the derivative:



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