Trigonometric Identities

Reciprocal Trigonometric Identities

$\sin\theta = 1/\csc\theta$ or $\csc\theta = 1/\sin\theta$

$\cos\theta = 1/\sec\theta$ or $\sec\theta = 1/\cos\theta$

$\tan\theta = 1/\cot\theta$ or $\cot\theta = 1/\tan\theta$

Pythagorean Trigonometric Identities

$\sin^2 a + \cos^2 a = 1$

$1+\tan^2 a = \sec^2 a$

$\csc^2 a = 1 + \cot^2 a$

Ratio Trigonometric Identities

$\tan\theta = \sin\theta/\cos\theta$

$\cot\theta = \cos\theta/\sin\theta$

Sum and Difference of Angles Trigonometric Identities

$\sin(\alpha+\beta)=\sin(\alpha).\cos(\beta)+\cos(\alpha).\sin(\beta)$

$\sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$

$\cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

Derivation Formula

Product Rule

$(d/dx)(fg) = fg' + gf'$

Quotient Rule

$(d/dx)(f/g) = gf'-fg'/g^2$

Chain Rule

$y = f(g(x))$, then $y' = f'(g(x)). g'(x)$

Derivatives of Trigonometric Functions

If $f(x) = \sin x$, then $f'(x) = \cos x$

If $f(x) = \cos x$, then $f'(x) = -\sin x$

If $f(x) = \tan x$, then $f'(x) = \sec 2 x$

If $f(x) = \cot x$, then $f'(x) = -\csc 2 x$.

If $f(x) = \sec x$, then $f'(x) = \sec x \tan x$

If $f(x) = \csc x$, then $f'(x) = -\csc x \cot x$

Example 1

$$f(x) = \tan x$$
$$= \frac{\sin x}{\cos x}$$
$$f'(x) = \frac{\cos x \cdot \cos x - (-\sin x)\sin x}{\cos^2 x}$$
$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$
$$= \frac{1}{\cos^2 x}$$
$$= \sec^2 x$$

Example 2

$$y' = 3x^2 \cot x + x^3(-\csc^2 x)$$
$$= 3x^2 \cot x - x^3 \csc^2 x$$

Examples

$g(x) = 3\sec(x) - 10\cot(x)$

Derivative of sin x

$$\frac{d(\sin x)}{dx} = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{(x+h)-x}$$
$$= \lim_{h\to 0}\frac{\sin x\cos h + \cos x\sin h - \sin x}{h}$$
$$= \lim_{h\to 0}\frac{\cos h - 1}{h}\sin x + \frac{\sin h}{h}\cos x$$
$$= (0)\sin x + (1)\cos x$$
$$= \cos x$$

Derivative of cos x

Derivative of sec x (cont)

$\sec x = 1/\cos x$

$\tan x = \sin x/\cos x$

$(\cos x)' = -\sin x$

$(\sec x)' = (1/\cos x)' = (-1/\cos 2x).(\cos x)'$

$= (-1/\cos 2x).(-\sin x)$

$= \sin x/\cos 2x$

$= (\sin x/\cos x).(1/\cos x)$

$= \tan x \sec x$

Therefore, $d(\sec x)/dx = \tan x \sec x$

Derivative of cot x

We will determine the derivative of cot x using the quotient rule. We will use the following formulas and identities to calculate the derivative:

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$\cot x = \cos x/ \sin x$

$\cos 2x + \sin 2x = 1$

$\csc x = 1/\sin x$

$(\cot x)' = (\cos x/\sin x)'$

$= [(\cos x)' \sin x - (\sin x)' \cos x]/\sin 2x$

$= [-\sin x. \sin x - \cos x. \cos x]/\sin 2x$

$= (-\sin 2x - \cos 2x)/\sin 2x$

$= -1/\sin 2x$

$= -\csc 2x$

Therefore, $d(\cot x)/dx = -\csc 2x$

Derivative of cosec x

We will determine the derivative of cosec x using the chain rule. We will use the following formulas and identities to calculate the derivative:

$\csc x = 1/\sin x$

$\cot x = \cos x/\sin x$

$(\sin x)' = \cos x$

$(\csc x)' = (1/\sin x)' = (-1/\sin 2x).(\sin x)'$

$= (-1/\sin 2x).(\cos x)$

$= -\cos x/\sin 2x$

$= -(\cos x/\sin x).(1/\sin x)$

$= -\cot x \csc x$

Therefore, $d(\csc x)/dx = -\cot x \csc x$

$$\frac{\mathrm{d}(\cos x)}{\mathrm{d}x} = \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{(x+h) - x}$$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos h - 1}{h} \cos x - \frac{\sin h}{h} \sin x$$

$$= (0) \cos x - (1) \sin x$$

$$= -\sin x$$

Derivative of sec x

We will determine the derivative of sec x using the chain rule. We will use the following formulas and identities to calculate the derivative:

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