

Old Exam

1. The data used in creating the attached one page of regression analysis printout come from a study of a nitride etch process on a single wafer plasma etcher. The process variables studied were

- x_1 = power applied to the cathode (W)
- x_2 = pressure in the reaction chamber (mTorr)
- x_3 = gap between the anode and the cathode (cm)
- x_4 = flow of the reactant gas C_2F_6

and the response variable was

y = selectivity of the process (SiN/poly silicon).

Note that the "StdErr Pred y " column on the output is for the second regression analysis.

Use the first regression analysis output in answering the questions (a)-(d) below. Note that $T_3 = 0.9636$ and $\sum(x_{3i} - \bar{x}_3)^2 = 0.3055$.

(a) What fraction of observed raw variation in y is explained by a linear equation in x_3 ?

[2] $R^2 = 0.799$

(b) What is the sample correlation between y and x_3 ?

[2] $-\sqrt{R^2} = -0.894$ (same sign as that of b_3)

(c) Give a 95% upper confidence bound for the increase in mean value of selectivity of the process for a 0.2 cm increase in gap between the anode and the cathode. (No need to simplify.)

[4] $0.2(b_3 + t_{\alpha/2, n-4} \frac{S_{E\hat{y}}}{\sqrt{\sum(x_{3i} - \bar{x}_3)^2}}) = 0.2(-1.096 + 1.833 \frac{0.0787}{\sqrt{0.3055}})$
 Q(.95) of t_6 is 1.833

(d) Give a 95% two-sided prediction interval for the next selectivity of the process when gap between the anode and the cathode equals 1.1 cm. (No need to simplify.)

[4] $\hat{y} \pm t_{\alpha/2, n-4} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum(x_{3i} - \bar{x}_3)^2}}$ Q(.975) of t_6 is 2.447
 $2.524 - 1.096(1.1) \pm 2.262 \cdot 0.0967 \sqrt{\frac{1}{7} + \frac{(1.1 - 0.9636)^2}{0.3055}}$

Old Exam 2

The following questions (e)-(j) are based on the multiple linear regression (MLR) model

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + \epsilon_i$$

Use the attached printout in answering these questions.

(e) What is the sample correlation between y and \hat{y} ?

[2] $\sqrt{R^2} = \sqrt{0.964} = 0.982$

(f) Give the fitted value and residual corresponding to the third observation (with $x_1 = 275, x_2 = 550, x_3 = 1.2, x_4 = 200$ and $y = 1.10$). (No need to simplify.)

[4] $\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4$
 $= 2.2496 + (0.00227)(275) + (-0.00132)(550) + (-1.0412)(1.2) + (-0.002532)(200)$
 $e = y - \hat{y} = 1.10 - \hat{y}$

(g) Give a 95% two-sided prediction interval for the next selectivity of the process corresponding to the conditions of the third observation. (No need to simplify.)

[4] $\hat{y} \pm t_{\alpha/2, n-4} \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\sum(x_{3i} - \bar{x}_3)^2}}$ Q(.975) of t_6 is 2.447
 $\hat{y} \pm 2.447 \sqrt{(0.0510)^2 + (0.0923)^2}$
 (\hat{y} is given in part (f) above)

(h) Give a 95% lower confidence bound for β_3 . (No need to simplify.)

[4] $b_3 - t_{\alpha/2, n-4} (S_{E\hat{\beta}_3})$ Q(.95) of t_6 is 1.833
 $-1.041 - (1.833) \cdot (0.0953)$

Old Exam 3

2. Consider a discrete random variable X with the following probability function.

x	-5	-3	-1	1	3	5
$f(x)$.1	.1	.3	.3	.1	.1

(a) Find the cumulative probability function, $F(x)$, for X .

[3] $F(x) = \begin{cases} 0 & x < -5 \\ .1 & -5 \leq x < -3 \\ .2 & -3 \leq x < -1 \\ .5 & -1 \leq x < 1 \\ .8 & 1 \leq x < 3 \\ .9 & 3 \leq x < 5 \\ 1 & x \geq 5 \end{cases}$

(b) Compute the mean and standard deviation of X .

[2] $EX = \sum x f(x) = (-5)(.1) + (-3)(.1) + (-1)(.3) + 1(.3) + 3(.1) + 5(.1) = 0$
 $Var X = E(X - EX)^2 = \sum (x - EX)^2 f(x) = (-5)^2(.1) + (-3)^2(.1) + (-1)^2(.3) + 1^2(.3) + 3^2(.1) + 5^2(.1) = 7.4$
 $\sqrt{Var X} = \sqrt{7.4} = 2.72$

Old Exam 4

(i) Find the value of an F statistic and its degrees of freedom for testing whether all the predictors x_1, x_2, x_3 , and x_4 can be dropped from this MLR model. What is your conclusion?

[4] Observed $F = \frac{90.21}{4} = 22.55$ df = 3, 6
 Conclusion (circle only one):
 (a) all the predictors should be dropped
 (b) not all the predictors should be dropped

(j) Find the value of an F statistic and its degrees of freedom for testing whether the predictors x_1, x_2 , and x_4 can be dropped from this MLR model. What is your conclusion?

[4] $F = \frac{(SSR_2 - SSR_4)/p}{SSE_4/(n-k-1)} = \frac{(9.9422 - 0.3394)/3}{0.0507/(11-4-1)} = 9.19$
 Observed $F = 9.19$ df = 3, 6
 Conclusion (circle only one):
 (a) x_1, x_2 , and x_4 should be dropped
 (b) at least one of x_1, x_2 , and x_4 should not be dropped

Old Exam 5

The following questions (e)-(j) are based on the multiple linear regression (MLR) model

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(e) What is the sample correlation between y and \hat{y} ?

[2] $\sqrt{R^2} = \sqrt{0.964} = 0.982$

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(g) Give a 95% two-sided prediction interval for the next selectivity of the process corresponding to the conditions of the third observation. (No need to simplify.)

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