Arithmetic Progression- Direct Formulae

1. Common Difference (d)= t_n-t_{n-1}
2. T_n= a+(n-1)d
3. Average method of arithmetic progression= (First term+Last term)/2= middle term=> (Sum of AP)/n
4. Sum (based on the previous line) = middle term x n
5. S_n= [n(2a+(n-1)d)/2]
6. Three numbers in AP are taken as a-d, a, a+d. Four numbers in AP are taken as a-3d, a-d, a+d, a+3d. Five numbers in AP are taken as a-2d, a-d, a, a+d, a+2d.
7. Inserting some numbers between two numbers to form an AP will lead to the total numbers being n+2.
8. Suppose P is the first term and Q is the last term=> Q is the (n+2)th term=> Q=P+(n+1)d
9. Deriving from the above sentence, d=(Q-P)/(n+1), required means (terms in the middle) are from [a+(b-a)/n-1] to [a+n(b-a)/n+1]

Arithmetic Progression- Indirect Tricks

1. In order to find the nth term of the sequence, add common difference to the first term, n-1 times
2. Every AP has an average. And for any AP, the average of any pair of corresponding terms will also be the average of the AP.
3. The summ of the term numbers for the terms of corresponding pairs is one greater the number of terms in an AP
4. Sum of AP= Number of terms x Average
5. Difference in the term numbers will give you the number of times the common differnce is uesed o the other to get from one to the other term.

Increasing and Decreasing AP

1. Increasing AP-	Every term of an increasing AP is greater than the previous term.
Case I:	When the first term of the increasing AP is positive
Case II:	When the first term of the increasing AP is negative. In this case there is a possibility for a sum till n_1 terms being the same and equal to the sum till n_2 terms. This case occurs when there is a balance about the number zero. The sum of th eterm numbers exhibiting equal sums is constant for a given AP. Additionally, when 0 is a part of the series, the sum can be equal to terms such that one of them is odd and the other is even, while when 0 is not a part of the series, the sum is equal for two terms when both of them are odd or even.
2. Decreasing AP-	Every term of a decreasing AP is lesser than the previous term
Case I:	Decreasing AP with first term negative
Case II:	Decreasing AP with first term positive

Geometric Progressions

1. Quantities are said to be in GP when they increase or decrease by a common factor- called common ratio.
2. Last term of the GP= ar^{n-1}
3. When choosing three numbers in GP, we take a/r, a and ar, and the middle one is the geometric mean of the other two. Four numbers in a GP can be taken as a/r^3, a/r,a, ar^3
4. Geometric mean otherwise= underroot(ab)
5. Inserting a number of means between two terms of a GP yields a series of n+2 terms. The common ratio in this case is r= $(b/a)^{(1/n+1)}$
6. Sum of numbers in GP=(if r>1) S_n= $a(r^n-1)/(r-1)$, while (if r<1), S_n=$a(1-r^n)/(1-r)$
7. Sum of an infinite progression= $S_{infinity}$= a/(1-r) {common ratio of the GP<1)

Increasing/Decreasing GP

Increasing GPs	Case I: First term positive and common ratio>1
	Case II: First term negative and common ratio<1
Decreasing GPs	Case I: First term positive and common ratio<1
	Case II: First term negative and common ratio>1

Important Series:

Sum of first n natural numbers:	n(n+1)/2
Sum of squares of first n natural numbers	{n(n+1)2n+1)/2}
Sum of cubes of first n natural numbers:	$[n(n+1)/2]^2$

Important Series: (cont)

Sum of first n odd natural numbers:	n^2
Sum of first n even natural numbers:	n^2+n
Sum of odd numbers <=n where n is a natural number:	Case A: If n is odd=> $[(n+1)/2]^2$
	Case B: If n is even=>$[n/2]^2$
Sum of even numbers<=n where n is a natural number:	Case A: If n is even=> n/2[(n/2)+1]
	Case B: If n is odd=> [(n-1)/2]-[(n+1)/2]

AP Type Series

1. First order series- nth term= (2n+1)	T_n= an+b
2. Second order series- nth term= (n^2+2n)	T_n= an^2+bn+c
3. Third order series- nth term= (n^3+n)	T_n= an^3+bn^2+cn+d

Harmonic Progression

1. If a,b,c,d are in AP, then 1/a,1/b,1/c and 1/d are in HP. In general three quantities a,b,c can be said to be in harmonic progression when a/c= (a-b)/(b-c)

2. There is no general formula for the sum of any number of quantities in harmonic progression.

3. The harmonic mean of any two given quantities (H)= 2ab/(a+b)

4. The nth term of a harmonic progression is given by- T_n= 1/(a+(n-1)d)

Theorems and Results- Progressions.

1. Given that A, G and H are the arithmetic, geometric and harmonic means respectively between a and b=> A= (a+b)/2, G=underroot(ab), H=2ab/(a+b). Therefore, this further implies that AxH=G^2 (that is G is the geometric mean between A and H). These results show that A-G= $[(\text{underroot}(a)-\text{underroot}(b))/\text{underroot}(2)]^2$ which is positive if a and b is positive. Therefore the arithmetic mean of any two quanitites is greater than the geometric mean.

2. A>G>H (The arithmetic, geometric and harmonic means between any two quantities are in descending order of magnitude

3. Combination of AP and GP=> It is a sequence of the form a, (a+d)r, (a+2d)r^2...where $T_n=[a+(n-1)d]r^{(n-1)}$

4. If the same quantity can be added or subtracted to or from all the terms of an AP, the resulting terms willl form an AP, but with the same common difference as before.

5. If all the terms of an AP can be multiplied or divided by the same quantity, the resulting terms will form an AP but with a new common difference, which will be the multiplication/division of the old common differenc

6. If all the terms of a GP are multiplied/divided by the same quantity, the resulting terms will form a GP with the same common ratio as before.

7. If a,b,c,d are in GP, they are also in continued proportion by definition= a/b=b/c=c/d=1/r. Conversely a series of quanitites in continued proportion can be represented in a geometric progression= a, ar, ar^2

Results and Theorems

Number of Terms in a count:

1. In general, if we are counting in steps of x from n_1 to n_2, including both end points, then we get $[(n_2-n_1)/x+1]$ numbers

2. For the aforementioned if we include only one end, we get $[(n_2-n_1)/x]$ numbers

3. If w exclude both ends for the same, we get $[(n_2-n_1)/x-1]$ numbers.

Note: An appropriate adjustment would have to be made if n_2 does not fall in the series- by taking the lower integral values in the answers.