

Turing Machine Basics

Transitions are in form: $a \rightarrow b, L$
meaning read a, write b, move left

Accept input if machine halts in an accept state

Reject input if machine halts in a non-accept state **or** machine enters an infinite loop

Turing Machines Definition

Turing Machine: $M = (Q, \Sigma, \Gamma, \delta, q_0, \emptyset, F)$

Q: States $\{q_0, q_1, q_2\}$

Σ : Input Alphabet $\{a, b\}$

Γ : Tape Alphabet $\{a, b\}$

δ : Transition functions $\delta(q_1, c) = (q_2, d, L)$

q_0 : Initial state

\emptyset : Blank

F: Accept States $\{q_2\}$

Deciders are Turing Machines that halt on all strings: always either accept or reject an input, never loop (infinitely)

Recognizers are Turing Machines that will halt and accept the strings in the language and either reject or do not halt for strings not in the language

Computing Functions with TM

Use unary (number represented as 1s e.g. $5 = 11111$)

Initial and Final configurations are at the beginning of the tape

Stay-Option TM

Head can move Left, Right or Stay

Stay-Option machines simulate Standard Turing machines (just don't use the stay)

Standard Turing machines simulate Stay-Option machines: just change the Stay transitions from

$a \rightarrow b, S$ to

$a \rightarrow b, L$ and $x \rightarrow x, R$ where $x \in \Gamma$

Semi-Infinite Tape TM

The head extends infinitely only to the right

Standard Turing machines simulate Semi-Infinite machines

Insert special symbol # on the left of the input string and add a selfloop to every state of $\# \rightarrow \#, R$

Semi-Infinite tape machines simulate Standard Turing machines: Squeeze infinity of both directions in one direction

Multi-Tape TM

Input string appears on Tape 1, but both tapes are read/write

Transitions are in form: $(b, f) \rightarrow (g, d), L, R$

Multi-tape machines simulate standard Turing Machines: The second one just remains empty

Standard Turing machines simulate Multi-tape machines: Uses a multi-track tape to simulate the multiple tapes

Multidimensional TM

For example 2-dimensional tape, has moves L, R, U, D (Up, down) and position of x and y

Multidimensional machines simulate Standard Turing machines: only use 1 dimension

Standard Turing machines simulate Multidimensional machines: Two tape machine

- One tape with two tracks (symbols in track 1, coordinates in track 2)

- Second tape for current coordinates

Nondeterministic TM

Nondeterministic machines simulate Standard (deterministic) Turing machines

Standard (deterministic) Turing machines simulate Nondeterministic machines:

- Use a 2D Tape

- Store all possible computations of the non-deterministic machine on the 2D Tape

