

### Turing Machine Basics

Transitions are in form:  $a \rightarrow b, L$   
meaning read a, write b, move left

Accept input if machine halts in an accept state

Reject input if machine halts in a non-accept state **or** machine enters an infinite loop

### Turing Machines Definition

Turing Machine:  $M = (Q, \Sigma, \Gamma, \delta, q_0, \emptyset, F)$

**Q:** States  $\{q_0, q_1, q_2\}$

**$\Sigma$ :** Input Alphabet  $\{a, b\}$

**$\Gamma$ :** Tape Alphabet  $\{a, b\}$

**$\delta$ :** Transition functions  $\delta(q_1, c) = (q_2, d, L)$

**$q_0$ :** Initial state

**$\emptyset$ :** Blank

**F:** Accept States  $\{q_2\}$

**Deciders** are Turing Machines that halt on all strings: always either accept or reject an input, never loop (infinitely)

**Recognizers** are Turing Machines that will halt and accept the strings in the language and either reject or do not halt for strings not in the language

### Computing Functions with TM

Use unary (number represented as 1s e.g.  $5 = 11111$ )

Initial and Final configurations are at the beginning of the tape

### Stay-Option TM

Head can move Left, Right or Stay

Stay-Option machines simulate Standard Turing machines (just don't use the stay)

Standard Turing machines simulate Stay-Option machines: just change the Stay transitions from

$a \rightarrow b, S$  to

$a \rightarrow b, L$  and  $x \rightarrow x, R$  where  $x \in \Gamma$

### Semi-Infinite Tape TM

The head extends infinitely only to the right

Standard Turing machines simulate Semi-Infinite machines

Insert special symbol # on the left of the input string and add a self-loop to every state of  $\# \rightarrow \#, R$

Semi-Infinite tape machines simulate Standard Turing machines: Squeeze infinity of both directions in one direction

### Multi-Tape TM

Input string appears on Tape 1, but both tapes are read/write

Transitions are in form:  $(b, f) \rightarrow (g, d), L, R$

Multi-tape machines simulate standard Turing Machines: The second one just remains empty

Standard Turing machines simulate Multi-tape machines: Uses a multi-track tape to simulate the multiple tapes

### Multidimensional TM

For example 2-dimensional tape, has moves L, R, U, D (Up, down) and position of x and y

Multidimensional machines simulate Standard Turing machines: only use 1 dimension

Standard Turing machines simulate Multidimensional machines: Two tape machine

- One tape with two tracks (symbols in track 1, coordinates in track 2)

- Second tape for current coordinates

### Nondeterministic TM

Nondeterministic machines simulate Standard (deterministic) Turing machines

Standard (deterministic) Turing machines simulate Nondeterministic machines:

- Use a 2D Tape

- Store all possible computations of the non-deterministic machine on the 2D Tape

