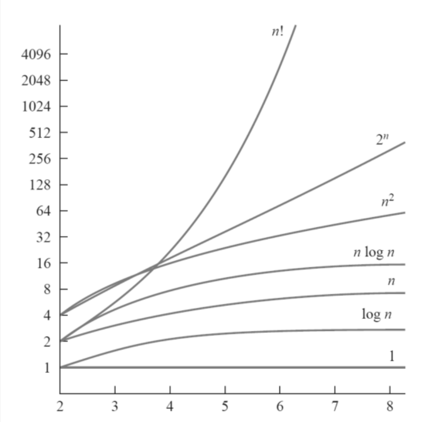
## Runtime Complexity




Formally: there exist constants c and n0 such that for all
sufficiently large n: f(n) ≤ c . g(n)
c,n0 n : n ≥ n0, f(n) ≤ c . g(n)

## Master theorem

T(n) = a·T(n/b) + f(n), a≥1 and b>1
let c = log_b a
Case 1: (only leaves) if f(n) = O($n^{c-e}$), then T(n) = Theta($n^c$) for some e>0
Case 2: (all nodes) if f(n) = theta($n^c \log^k n$) , k≥0 , T(n) = theta($n^c \log^{k+1} n$)
Case 3: (only internal nodes) if f(n) = omega($n^{c+e}$), then T(n) = theta(f(n)) for some e>0

## Kruskals Algorithm

Sort all edges by their weights
Loop:
- Choose the minimum weight edge and join correspondent vertices (subject to cycles).
- Go to the next edge.
- Continue to grow the forest until all vertices are connected

Runtime Complexity:
Sorting edges – O(E log E)
Cycle detection – O(V) for each edge
Total: O(V * E + E * log E)

## Depth-First-Search (DFS)

It starts at a selected node and explores as far as possible along each branch before backtracking. DFS uses a stack for backtracking

## Breadth-First-Search (BFS)

It starts at a selected node and explores all nodes at the present depth prior to moving on to the nodes at the next depth level. BFS uses a FIFO queue for bookkeeping

## Amortized Analysis

Aggregate method: The amortized cost of an operation is given by $T(n) / n$
Accounting Method: We assign different charges to each operation; some operations may charge more or less than they actually cost.

## Topological Sort

1. Select a vertex that has zero in-degree.
2. Add the vertex to the output.
3. Delete this vertex and all its outgoing edges.
4. Repeat

## Coin Change

opt[k,x] = min(opt[k-1,x] , opt[k,x - dk] + 1)
Base : opt[1,x] = x, opt[k,0] = 0

## 01 knapsack

opt[k,x] = max(vk + opt[k-1, x - wk], opt[k-1,x]
base: opt[0,x] = 0, opt[k,0] = 0
opt[k,x] = opt[k-1,x] if wk > x

## Djikstra's Algorithm

When algorithm proceeds, all vertices are divided into two groups
- vertices whose shortest path from the source is known
- vertices whose shortest path from the source is NOT discovered yet.

Move vertices one at a time from the undiscovered set of vertices to the known set of the shortest distances, based on the shortest distance from the source.
Runtime: O(V.log V + E.log V)

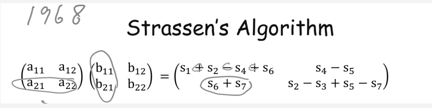
## Heap

| | Binary | Binomial | Fibonacci |
|---|---|---|---|
| findMin | $\Theta(1)$ | $\Theta(1)$ | $\Theta(1)$ |
| deleteMin | $\Theta(\log n)$ | $\Theta(\log n)$ | $O(\log n)$ (ac) |
| insert | $\Theta(\log n)$ | $\Theta(1)$ (ac) | $\Theta(1)$ |
| decreaseKey | $\Theta(\log n)$ | $\Theta(\log n)$ | $\Theta(1)$ (ac) |
| merge | $\Theta(n)$ | $\Theta(\log n)$ | $\Theta(1)$ (ac) |

## Karatsuba

a × b = $(x1 \cdot 10^{n/2} + x0) \cdot (y1 \cdot 10^{n/2} + y0)$

## Strassen Algorithm




## Prim's Algorithm

1) Start with an arbitrary vertex as a sub-tree C.
2) Expand C by adding a vertex having the minimum weight edge of the graph having exactly one end point in C.
3) Update distances from C to adjacent vertices.
4) Continue to grow the tree until C gets all vertices.

Runtime:
binary heap : O(V.log V + E. log V)
Fibonacci heap: O(V. log V + 1) (ac)

## Greedy Algorithm

It is used to solve optimization problems
It makes a local optimal choice at each step
Earlier decisions are never undone
Does not always yield the optimal solution

## Longest Common Subsequence

LCS[i,j] = (1 + LCS[i-1,j-1]) if s[i] = s[j]
LCS[i,j] = (max(lcs[i-1,j] , max[i,j-1]) if s[i] != s[j]
base case: lcs[i,0] = lcs[0,j] = 0