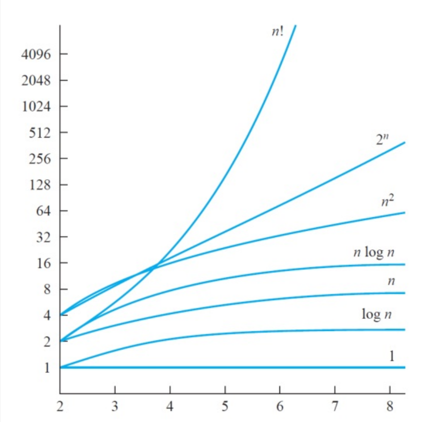
Runtime Complexity



Formally: there exist constants c and n0 such that for all sufficiently large n: f(n) ≤ c . g(n)

c,n0 n : n ≥ n0, f(n) ≤ c . g(n)

Master theorem

T(n) = a·T(n/b) + f(n), a≥1 and b>1

let c = log_b a

Case 1: (only leaves) if f(n) = O(n^{c-e}), then T(n) = Theta(n^c) for some e>0

Case 2: (all nodes) if f(n) = theta(n^c $\log^k$ n) , k≥0 , T(n) = theta(n^c $\log^{k+1}$n)

Case 3: (only internal nodes) if f(n) = omega(n^{c+e}), then T(n) = theta(f(n)) for some e>0

Kruskals Algorithm

Sort all edges by their weights

Loop:

- Choose the minimum weight edge and join correspondent vertices (subject to cycles).
- Go to the next edge.
- Continue to grow the forest until all vertices are connected

Runtime Complexity:

Sorting edges – O(E log E)

Cycle detection – O(V) for each edge

Total: O(V * E + E * log E)

Depth-First-Search (DFS)

It starts at a selected node and explores as far as possible along each branch before backtracking. DFS uses a stack for backtr-acking

Breadth-First-Search (BFS)

It starts at a selected node and explores all nodes at the present depth prior to moving on to the nodes at the next depth level. BFS uses a FIFO queue for bookkeeping

Amortized Analysis

Aggregate method: The amortized cost of an operation is given by $T(n) / n$

Accounting Method: We assign different charges to each operation; some operations may charge more or less than they actually cost.

Topological Sort

1. Select a vertex that has zero in-degree.
2. Add the vertex to the output.
3. Delete this vertex and all its outgoing edges.
4. Repeat

Coin Change

opt[k,x] = min(opt[k-1,x] , opt[k,x - dk] + 1)

Base : opt[1,x] = x, opt[k,0] = 0

01 knapsack

opt[k,x] = max(vk + opt[k-1, x - wk], opt[k-1,x]

base: opt[0,x] = 0, opt[k,0] = 0

opt[k,x] = opt[k-1,x] if wk > x

Djikstra's Algorithm

When algorithm proceeds, all vertices are divided into two groups

- vertices whose shortest path from the source is known
- vertices whose shortest path from the source is NOT discovered yet.

Move vertices one at a time from the undiscovered set of vertices to the known set of the shortest distances, based on the shortest distance from the source.

Runtime: O(V.log V + E.log V)

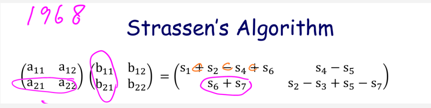
Heap

	Binary	Binomial	Fibonacci
findMin	Θ(1)	Θ(1)	Θ(1)
deleteMin	Θ(log n)	Θ(log n)	O(log n) (ac)
insert	Θ(log n)	Θ(1) (ac)	Θ(1)
decreaseKey	Θ(log n)	Θ(log n)	Θ(1) (ac)
merge	Θ(n)	Θ(log n)	Θ(1) (ac)

Karatsuba

a × b = $(x1 \cdot 10^{n/2} + x0) \cdot (y1 \cdot 10^{n/2} + y0)$

Strassen Algorithm



Prim's Algorithm

1) Start with an arbitrary vertex as a sub-tree C.
2) Expand C by adding a vertex having the minimum weight edge of the graph having exactly one end point in C.
3) Update distances from C to adjacent vertices.
4) Continue to grow the tree until C gets all vertices.
Runtime:
binary heap : O(V.log V + E. log V)
Fibonacci heap: O(V. log V + 1) (ac)

Greedy Algorithm

It is used to solve optimization problems
It makes a local optimal choice at each step
Earlier decisions are never undone
Does not always yield the optimal solution

Longest Common Subsequence

LCS[i,j] = (1 + LCS[i-1,j-1]) if s[i] = s[j]
LCS[i,j] = (max(lcs[i-1,j] , max[i,j-1]) if s[i] != s[j]
base case: lcs[i,0] = lcs[0,j] = 0