

Matrices

Addition	$X + Y = [z_{ij}] = [x_{ij} + y_{ij}]$
Subtraction	$X - Y = [z_{ij}] = [x_{ij} - y_{ij}]$
Multiplication	$X * Y = [z_{ij}] = [x_i * y_j]$
Constant	$c * X = [z_{ij}] = [c * x_{ij}]$

Transpose & Identity

Transpose	$X^T = [z_{ij}] = [x_{ji}]$
Tr of Tr	$(X^T)^T = X$
Tr of Mul	$(XY)^T = Y^T X^T \neq X^T Y^T$
Sym Matrix	$X^T = X$
Identity Matrix I	$X I = I X = X$ [z _{ii} =1, z _{ij} =0]

Inverse

Inverse	$X X^{-1} = I = X^{-1} X$
if X^{-1} exists then X is non singular or invertible	
Inv of Inv	$(X^{-1})^{-1} = X$
Inv of Mul	$(XY)^{-1} = Y^{-1} X^{-1} \neq X^{-1} Y^{-1}$
Inv of Tr	$(X^T)^{-1} = (X^{-1})^T$
Determinant	$ A = \sum_{i=1}^n a_{ij} \times \text{Det } a_{ij} $
Determinant is computed over first row of matrix where each element of first row is multiplied by its minor	
minor M_{ij} is a determinant obtained by deleting the i^{th} row and j^{th} column in which a_{ij} lies. Minor of a_{ij} is denoted by m_{ij} .	
Cofactor	$A_{ij} = (-1)^{i+j} m_{ij}$
Adjoint	$\text{adj}(A) = (\text{Cofactor})^T = (A_{ij})^T$
Inverse	$A^{-1} = \text{adj}(A) / A $

Orthogonal

Two $n \times 1$ vectors are orthogonal if $X^T Y = 0$	
A vector is orthonormal if $X^T X = X^2 $	
Sq root of $ X $ is length or norm of vector	
{ $X_1, X_2, X_3 \dots X_n$ } are said to be orthonormal if, each pair is orthogonal and have unit length	
A sq matrix is orthogonal if $X^T X = I$ or $X^T = X^{-1}$	

Eigen Values & Eigen Vectors

A is $n \times n$ matrix, X is $n \times 1$ matrix, λ is a scalar, then	
$AX = \lambda X$ or $(A - \lambda I)X = 0$ or $X = (A - \lambda I)^{-1}$	
λ is the eigen value and X is the eigen vector (non zero)	
Since X is non zero, $ A - \lambda I $ should be 0	
Determinant for $[a \ b] = ad - bc$ [c d]	
If $A \Rightarrow$ symmetric, then eigenvalues $\Rightarrow$ real & eigenvectors $\Rightarrow$ orthogonal	
Diagonalization: $P \Rightarrow$ orthogonal matrix, then $Z = P^T A P$, Z is diagonal matrix with eigen values of A	

Linear Independence

Given $a_1 x_1 + a_2 x_2 + \dots a_n x_n = 0$, if a vector $[a_1, a_2, \dots a_n]$ exists such that	
a. all a_i are 0, then x_i are linearly independent.	
b. if some $a_i \neq 0$ then x_i are linearly dependent.	
If a set of vectors are linearly dependent, then one of them can be written as some combination of others	
A set of two vectors is linearly dependent if and only if one of the vectors is a constant multiple of the other.	

Idempotence

a $n \times n$ matrix A is idempotent iff $A^2 = A$	
The identity matrix I is idempotent.	
Let X be an $n \times k$ matrix of full rank, $n \geq k$ then H exists as $H = X(X^T X)^{-1} X^T$ and is idempotent.	

Rank

For a $n \times k$ matrix say X, the column vectors are $[x_1, x_2, \dots x_k]$ and rank is given by max num of linearly independent vectors.	
If X is a $n \times k$ matrix and $r(X) = k$, then X is of full rank for $n \geq k$.	
$r(X) = r(X^T) = r(X^T X)$	
If X is $k \times k$, then X is non singular iff $r(X) = k$.	
If X is $n \times k$, P is $n \times n$ and non-singular, and Q is $k \times k$ and nonsingular, then $r(X) = r(PX) = r(XQ)$.	
The rank of a diagonal matrix is equal to the number of non zero diagonal entries in the matrix.	
$r(XY) \leq r(X) \ r(Y)$	

Trace

The trace of a square $k \times k$ matrix X is sum of its diagonal entries -	
$\text{tr}(X) = \sum x_{ii}$	
If c is a scalar, $\text{tr}(cX) = c * \text{tr}(X)$	
$\text{tr}(X \pm Y) = \text{tr}(X) \pm \text{tr}(Y)$.	
If XY and YX both exist, $\text{tr}(XY) = \text{tr}(YX)$.	

Quadratic Forms

A be a $k \times k$, y be $k \times 1$ vector containing variables $q = y^T A y$ is called a quadratic form in y, A is called the matrix of the quadratic form	
$q = \sum \sum a_{ij} y_i y_j$	
If $y^T A y > 0$ for all $y \neq 0$, $y^T A y$ & A are +ve definite	
If $y^T A y \geq 0$ for all $y \neq 0$, $y^T A y$ & A are +ve semidefinite	

Matrix Differentiation

$y = (y_1, y_2, \dots, y_k)^T$, $z = f(y)$ then $\partial z / \partial y = [\partial z / \partial y_1 \ \partial z / \partial y_2 \ \partial z / \partial y_3]^T$	
$z = a^T y$, $\partial z / \partial y = a$	
$z = y^T y$, $\partial z / \partial y = 2y$	
$z = y^T A y$, $\partial z / \partial y = Ay + A^T y$, if A is symmetric then $\partial z / \partial y = 2Ay$	

Theorems

Theorem 1

Let A be a symmetric $k \times k$ matrix. Then an orthogonal matrix P exists such that $P^T A P = \Lambda$ where $\Lambda = [\lambda_1, \lambda_2, \dots, \lambda_n]$ are the eigen values of A as $n \times 1$ vector

Theorem 2

The eigenvalues of idempotent matrices are always either 0 or 1.

Theorem 3

If A is a symmetric and idempotent matrix,
 $r(A) = \text{tr}(A)$

Theorem 4

Let $A_1, A_2, \dots, A_m$ be a collection of symmetric $k \times k$ matrices. Then the following are equivalent:

- There exists an orthogonal matrix P such that $P^T A_i P$ is diagonal for all $i = 1, 2, \dots, m$;
- $A_i A_j = A_j A_i$ for every pair $i, j = 1, 2, \dots, m$.

Theorem 5

Let $A_1, A_2, \dots, A_m$ be a collection of symmetric $k \times k$ matrices. Then any two of the following conditions implies the third:

- All A_i , $i = 1, 2, \dots, m$ are idempotent;
- $\sum A_i$ is idempotent;
- $A_i A_j = 0$ for $i \neq j$

Theorem 6

Let $A_1, A_2, \dots, A_m$ be a collection of symmetric $k \times k$ matrices. If the conditions in Theorem 5 are true, then
 $r(\sum A_i) = \sum r(A_i)$

Theorem 7

A symmetric matrix A is positive definite if and only if its eigen values are all (strictly) positive

Theorem 8

A symmetric matrix A is positive semi-definite if and only if its eigenvalues are all non-negative.



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Page 2 of 2.

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