Matrices Cheat Sheet

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Matrices

Addition	X + Y = [zij] = [xij + yij]
Subtraction	X - Y = [zij] = [xij - yij]
Multiplication	X * Y = [zij] = [xi * yj]
Constant	c * X = [zij] = [c * xij]

Transpose & Identity

Transpose	X^T = [zij] = [xji]
Tr of Tr	$(X^T)^T = X$
Tr of Mul	$(XY)^T = Y^T X^T \ != X^T Y^T$
Sym Matrix	$X^T = X$
Identity Matrix I [zii=1, zij=0]	X *I* = *I* X = X

Inverse

Inverse	$X X^{-1} = I = X^{-1}X$
if X^{-1} exists then X is non singular or invertible	
Inv of Inv	$(X^{-1})^{-1} = X$
Inv of Mul	$(XY)^{-1} = Y^{-1}X^{-1} \ != X^{-1}Y^{-1}$
Inv of Tr	$(X^T)^{-1} = (X^{-1})^T$
Determinant	$\lvert A\rvert = {}^n\sum_{i=1}$ aij x Det |aij|
Determinant is computed over first row of matrix where each element of first row is multiplied by its minor	
minor Mij is a determinant obtained by deleting the i^{th} row and j^{th} column in which aij lies. Minor of aij is denoted by mij.	
Cofactor	Aij = $(-1)^{i+j}$ mij
Adjoint	adj(A) = $(\text{Cofactor})^T$ = $(\text{Aij})^T$
Inverse	A^{-1} = adj(A) / |A|

Orthogonal

Two n x 1 vectors are orthogonal if $X^T Y = 0$

A vector is orthonormal if $X^TX = ||X^2||$

Sq root of ||X|| is length or norm of vector

{X1, X2, X3.... Xn) are said to be orthonormal if, each pair is orthogonal and have unit length

A sq matrix is orthogonal if $X^TX= I$ or $X^T=X^{-1}$

Eigen Values & Eigen Vectors

A is nxn matrix, X is nx1 matrix, λ is a scalar, then

AX = λX or (A-λI)X = 0 or X = $(A-\lambda I)^{-1}$

λ is the eigen value and X is the eigen vector (non zero)

Since X is non zero, |A-λI| should be 0

Determinant for [a b] = ad - bc
[c d]

If A => symmetric, then eigenvalues => real &
eigenvectors => orthogonal

Diagonalization: P => orthogonal matrix, then Z = P^TAP, Z is diagonal matrix with eigen values of A

Linear Independence

Given a1x1 + a2x2 + ...anxn = 0, if a vector [a1, a2, ...an] exists such that

a. all ai are 0, then xi are linearly independent.

b. if some ai!=0 then xi are linearly dependent.

If a set of vectors are linearly dependent, then one of them can be written as some combination of others

A set of two vectors is linearly dependent if and only if one of the vectors is a constant multiple of the other.

Idempotence

a nxn matrix A is idempotent iff $A^2 = A$

The identity matrix I is idempotent.

Let X be an n×k matrix of full rank ,n≥k then H exists as $H=X(X^TX)^{-1}X^T$ and is idempotent.

Rank

For a nxk matrix say X, the column vectors are [x1, x2, ...xk] and ***rank*** is given by max num of linearly independent vectors.

If X is a nxk matrix and r(X) = k, then X is of full rank for n≥k.

$r(X) = r(X^T) = r(X^TX)$

If X is kxk, then X is non singular iff r(X) = k.

If X is n×k, P is n×n and non-singular, and Q is k×k and nonsingular, then r(X) =r(PX) =r(XQ).

The rank of a diagonal matrix is equal to the number of non zero diagonal entries in the matrix.

r(XY) ≤ r(X) r(Y)

Trace

The trace of a square k×k matrix X is sum of its diagonal entries -
tr(X) = ∑ xii

If c is a scalar, tr(cX) =c * tr(X)

tr(X±Y) =tr(X) ± tr(Y).

If XY and YX both exist, tr(XY) =tr(YX).

Quadratic Forms

A be a k × k, y be k × 1 vector containing variables q = y^T Ay is called a quadratic form in y, A is called the matrix of the quadratic form

q = ∑ ∑ aijyiyj

If $y^TAy > 0$ for all y != 0, y^TAy & A are +ve definite

If $y^TAy >= 0$ for all y != 0, y^TAy & A are +ve semidefinite

Matrix Differentiation

y = $(y1, y2, . . . , yk)^T$, z = f(y) then ∂z/∂y = $[∂z/∂y1\ ∂z/∂y2\ ∂z/∂y3]^T$

$z=a^Ty$, ∂z/∂y = a

$z=y^Ty$, ∂z/∂y = 2y

$z=y^TAy$, ∂z/∂y = $Ay+A^Ty$, if A is symmetrix then ∂z/∂y = 2Ay

Theorems

Theorem 1

Let A be a symmetric k×k matrix. Then an orthogonal matrix P exists such that $P^TAP = \lambda \times I$, where $\lambda = [\lambda_1, \lambda_2, ... \lambda_n]$ are the eigen values of A as nx1 vector

Theorem 2

The eigenvalues of idempotent matrices are always either 0 or 1.

Theorem 3

If A is a symmetric and idempotent matrix, r(A) =tr(A)

Theorem 4

Let $A_1, A_2, \ldots, A_m$ be a collection of symmetric k×k matrices. Then the following are equivalent:

a. There exists an orthogonal matrix P such that P^TA_iP is diagonal for all i= 1,2,...,m;

b. $A_iA_j = A_jA_i$ for every pair i,j= 1,2,...,m.

Theorem 5

Let $A_1, A_2, \ldots, A_m$ be a collection of symmetric k×k matrices. Then any two of the following conditions implies the third:

a. All Ai, i= 1,2,...,m are idempotent;

b. ∑ Ai is idempotent;

c. AiAj= 0for i6=j

Theorem 6

Let $A_1, A_2, \ldots, A_m$ be a collection of symmetric k×k matrices. If the conditions in Theorem 5 are true, then

$r(\sum A_i) = \sum r(A_i)$

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A symmetric matrix A is positive definite if and only if its eigen values are all (strictly) positive

Theorem 8

A symmetric matrix A is positive semi-definite if and only if its eigenvalues are all non-negative.