

List of Equivalences	
Conditional Statement	$p \rightarrow q \equiv \sim p \vee q$ $(p \wedge \sim q)$
contrapositive	$p \rightarrow q \equiv \sim q \rightarrow \sim p$
Converse	$p \rightarrow q$ (cond) $q \rightarrow p$ (converse)
inverse	$p \rightarrow q$ (cond) $\sim p \rightarrow \sim p$ (inverse)
vacuously true = true by absence of counterexamples converse and inverse are the SAME	

Useful Symbols	
\forall	for all (universal operator)
\exists	exists (existential operator)
\in	in the set
\wedge	and
\vee	or
\sim	not
\equiv	equivalent
\subset	subset
\supset	superset
$\{\}, \emptyset$	empty set
\leftrightarrow	biconditional (both are true)

Statements	
Universal	For all, for each
Existential	At least, there exists
Conditional	If \rightarrow then
Universal Conditional	For all & if-then

Statements (cont)	
Universal Existential	For all & there exists
Existential Universal	There exists & for all

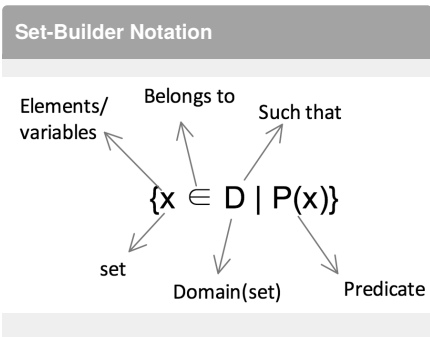
Functions

Requirements:

- Arrow coming out of **every element** in domain
- Every element can only have **one** element of *codomain*

unsatisfied requirement = relation
y can be used repeatedly but x values only have one arrow coming out

Predicates and Quantified Statements		
Statement type	original	negated
Universal	$\forall x \in D, P(x)$	$\exists x \in D, \sim P(x)$
Existential		
Universal Conditional		



Set-Roster Notation

$A = \{1, 2, 3 \dots 100\}$

use ellipses for larger sets

Subsets

$B \subseteq A$ $B = \text{subset}$,
 $A = \text{superset}$

Proper Subsets: elements that belong to superset but NOT subset

Relations

Relations = subsets of cartesian product

$R \subseteq A \times B$ Relation \subseteq Domain \times Codomain

Domain **SET** that includes every element from source

don't always have to include ordered pairs

DeMorgan's Law

- Tells us how to handle conjunction and disjunction negations
- $\sim(p \wedge q) \equiv \sim p \vee \sim q$
- $\sim(p \vee q) \equiv \sim p \wedge \sim q$
- "The connector is loose(l) or the machine is unplugged(u)"
- $\neg \vee \neg \equiv \neg \wedge$ $\neg \wedge \neg \equiv \neg \vee$
- "The connector is **not** loose *and* the machine is **not** unplugged"
- $\sim p \vee q$ is the opposite of $p \wedge \sim q$

When using DeMorgan's law, no need for truth table

Tautologies and Contradictions		
Tautologies	Always true statements	t
Contradictions	Always false statements	c
$p \wedge \sim p$	$\top \wedge \text{False}$	$\text{False} \wedge \top$
$p \vee \sim p$	$p \wedge c$	

Absorption law: variable absorbing operator
 \Rightarrow use truth table to prove law
 \Rightarrow other variables don't play a role in statement validity
 $p \vee (p \wedge q) \equiv p$; $p \wedge (p \vee q) \equiv p$

$p \rightarrow q$ truth table

Truth Table for $p \rightarrow q$		
p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Argument Truth Table

premises							conclusion	
p	q	r	$\sim p$	$\sim q$	$p \wedge r$	$p \rightarrow q$	$q \rightarrow p \wedge r$	$p \rightarrow r$
T	T	T	F	F	T	T	T	T
T	T	F	F	F	F	T	F	F
T	F	T	F	T	F	F	T	F
T	F	F	F	T	F	T	F	F
F	T	T	T	F	T	T	T	T
F	T	F	T	F	F	T	T	T
F	F	T	T	T	F	F	T	T
F	F	F	T	T	F	T	T	T

This row shows that an argument of this form, where premises and conclusion are true. Hence this argument is invalid.

Critical row = row where both premises are true
 premises and conclusion = TRUE is a **valid** argument

Arguments

$p \rightarrow q$	major premise
p	minor premise
$\therefore q$	therefore, conclusion

premises aka assumptions or hypotheses verified using truth table



Argument Forms (VALID)	
Modus Ponens	$p \rightarrow q$
	p
	$\therefore q$
Modus Tollens	$p \rightarrow q$
	$\sim q$
	$\therefore \sim p$
Generalization	p
	$\therefore p \vee q$
Specialization	$p \wedge q$
	$\therefore q$
Elimination	$p \vee q$
	$\sim q$
	$\therefore p$
Transitivity	$p \rightarrow q$
	$q \rightarrow r$
	$\therefore p \rightarrow r$
Proof by div. into cases	$p \vee q$
	$p \rightarrow r$
	$q \rightarrow r$
	$\therefore r$

Fallacy (INVALID ARGUMENTS)	
Converse Error	$p \rightarrow q$
	q
\Rightarrow	$\therefore p$
Inverse Error	$q \rightarrow p$
	$\sim p$
	$\therefore \sim q$

