Classical + Relative

$P(A) = N(A)/N(S)$
$P(A) = f(A)/n$

Conditional

$P(A|B) = P(A\cap B)/P(B)$

A given B

CDF

$F(x) = P(X\leq x) = \Sigma f(x\_i)$

Joint PMF

$p(x,y) = P(X=x, Y=y) = P(\{X=x\}\cap\{Y=y\})$

Geometric Distribution

X = # of trials until 1st success
$X \sim g(p)$
$f(x) = (1-p)^{x-1}p$, for x=1,2,...
$F(x) = 1-(1-p)^x$, for x=1,2,...
$E[X] = 1/p$
$V[X] = (1-p)/p^2$

Continuous Variable

$P(a<X<b) = \int f(x)dx = F(b)-F(a)$
$f(x) = F'(x)$
$F(x) = P(X<x) = \int f(t)dt$
$E[X] = \int xf(x)dx$
$V[X] = \int x^2f(x)dx-E[X]^2$
$E[g(X)] = \int g(x)f(x)dx$
$V[g(X)] = \int (g(x))^2f(x)dx=E[g(X)]^2$

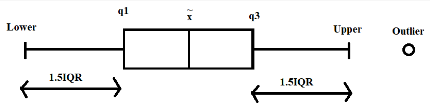
Normal Distribution

$f(x) = 1/\sqrt{(2\pi\sigma^2)}*e^{-(x-\mu)^2/(2\sigma^2)}$, $-\infty<x<\infty$
$X \sim N(\mu, \sigma^2)$
$E[X] = \mu$
$V[X] = \sigma^2$

Sample Mean

$\bar{x} = \Sigma x\_i/n$

Box Plot



Describe histogram: skewness, uni/bi-modal

Constructing Confidence Interval

$P = Y/n$
$Y \sim b(n,p)$
$Z = (P-p)/\sqrt{(p(1-p)n)} \sim N(0,1)$
$E = z\_[\alpha/2]\sqrt{(p(1-p)/n)}$

Sample Correlation

$r = cov/(s\_xs\_y)$

s_x and s_y are standard dev.

Permutations

$n! = n(n-1)(n-2)*...*1$ if $n\geq 1$
$= 1$ if $n=0$
$nPr = n!/(n-r)!$

Order matters

PMF

$f(x) = P(X=x)$

Variance

$\sigma^2 = V[X] = \Sigma x^2f(x)-E[X]^2$

Standard deviation = sqrt(V[X])

Joint Properties

$E[g(X,Y)] = \Sigma^x\Sigma^y g(x,y)p(x,y)$
$E[X] = \Sigma^x xp(x)$
$E[Y] = \Sigma^y yp(y)$
$E[X+Y] = E[X]+E[Y]$
$Cov[X,Y] = (\Sigma^x\Sigma^y xyp(x,y))-E[X]E[Y]$
$V[X+Y] = V[X]+V[Y]+2Cov[X,Y]$

Poisson Distribution

X = # of event in time [0,1]
$p(x) = e^{-\mu}*\mu^x/x!$, for x=0,1,...
$X \sim P(\mu)$
$E[X] = V[X] = \mu$
Approximation: binomial f(x) $\approx$ p(x), $\mu=np$
Process: between [0,t], $\mu=\lambda t$

Continuous Uniform Distribution

$f(x) = 1/(b-a)$, $a\leq x\leq b$
$= 0$, elsewhere
$X \sim U[a,b]$
$E[X] = (a+b)/2$
$V[X] = (b-a)^2/12$

Sample Variance

$s^2 = ((\Sigma x^2\_i)-n\bar{x}^2)/(n-1)$

CLT

$Z = (\bar{X}-\mu)/(\sigma/\sqrt{n})$
$\bar{X} \sim N(\mu, \sigma^2/n) \Rightarrow Z \sim N(0,1)$

Confidence Level

$\alpha = P(Z>z\_\alpha) = 1-\Phi(z)$
$\mu \in [\bar{x}-E, \bar{x}+E]$
σ^2 known: $E = z\_[\alpha/2]*\sigma/\sqrt{n}$
σ^2 unknown: $T = (\bar{X}-\mu)/(S/\sqrt{n}) \sim T(n-1)$
$P(T>t\_[\alpha,v]) = \alpha$; $z\_\alpha = t\_[\alpha,\infty]$
$E = t\_[\alpha/2,n-1]*s/\sqrt{n}$
σ^2 unknown, $n\geq 40$: $(\bar{X}-\mu)/(S/\sqrt{n}) \sim N(0,1)$
$E = z\_[\alpha/2]*s/\sqrt{n}$
$n\geq((z\_[\alpha/2]\sigma)/E)^2$

Combinations

$n = n\_1*...*n\_k$
$nCr = (^nr) = n!/r!(n-r)!$

Order doesn't matter

Multiplucation Rule

$P(A\cap B) = P(B|A)P(A) = P(A|B)P(B)$
$= P(A)P(B)$ if ind.

Transformation

$E[g(X)] = \Sigma g(x)f(x)$
$V[g(X)] = [\Sigma(g(x))^2f(x)]-(E[g(X)])^2$

Bernoulli Trial

S = {success, failure} = {p,q}
p = P(I=1)
I ~ Ber(p)
E[I] = p
V[I] = p(1-p)

Negative Binomial Distribution

X = # of trials to until r^{th} success
X ~ Nb(r,p)
f(x) = $(^{x-1}r-1)(1-p)^{x-r}p^r$, for x=r,r+1,...
E[X] = r/p
V[X] = $r(1-p)/p^2$

Erlang Distribution

T = time until r^{th} outcome of Poisson process
F(x) = P(T≤x) = 1-P(T>x)
= $1-\Sigma^{r-1}e^{-\lambda x}(\lambda x)^k/k!$
E[T] = r/λ
V[T] = $r(1-\lambda)/\lambda^2$

Standardization Thm

Z = (X-E[X])/√(V[X])
F(x) = P(X≤x) = Φ((x-μ)/σ)
P(a<X<b) = F(b)-F(a)

Percentile

Rank of k^{th} percentile: (n+1)*k/100 = m+p, 0≤p<1
k^{th} percentile = y_m+p(y_[m+1]-y_m)
IQR = q_3-q_1

Median is 50^{th} percentile

Hypothesis

Null hyp: make no change
Alternate hyp: test according to question
⇒Test 1: μ ≠ μ_0; 2: μ > μ_0; 3: μ < μ_0;
Confidence interval decision: reject H_0 for H_1 if μ_0 is not in confidence interval
Z_0 or T_0 decision:
σ^2 known: Z_0 = (X̄-μ_0)/(σ/√n) ~ N(0,1)
Test 1: reject if |z_0| > z_[α/2]; 2: z_0 > z_α; 3: z_0 < -z_α
σ^2 unknown: T_0 = (X̄-μ_0)/(S/√n) ~ T_[n-1]
Test 1: |t_0| > t_[α/2,n-1]; 2: t_0 > t_[α,n-1]; 3: t_0 < -t_[α,n-1]
Pop. & σ^2 unknown: replace σ with S from σ^2 known
p-Value decision: reject if p-value < α
p-value = 2[1-Φ(|z_0|)], test 1 & z-value
= 1-Φ(z_0), test 2 & z-value
= Φ(z_0), test 3 & z-value
= 2P(T>|t_0|), test 1 & t-value
= P(T>t_0), test 2 & t-value
= P(T<t_0), test 3 & t-value

Addition Rules

P(A∩B') = P(A)-P(A∩B)
P(A∪B) = P(A)+P(B)-P(A∩B)
P(A'∩B') = 1-P(A∪B)
P(A∪B∪C) = P(A)+P(B)+P(C)-P(A∩B)-P(A∩C)-P(B∩C)+P(A∩B∩C)
P(A_1∪...∪A_n) = 1-P(A_1'∩...∩A_n')

Expected Value

μ = E[X] = Σxf(x)

Marginal PMF

p(x) = P(X=x) = Σ^yp(x,y)
p(y) = P(Y=y) = Σ^xp(x,y)

Binomial Distribution

X = # of successes from n trials
X ~ b(n,p)
f(x) = $(^n x)p^x(1-p)^{n-x}$, for x=0,1,...,n
E[X] = np
V[X] = np(1-p)

Exponential Distribution

Waiting time
X ~ Exp(λ)
f(x) = $\lambda e^{-\lambda x}$, x>0
F(x) = $1-e^{-\lambda x}$, x>0
E[X] = 1/λ
V[X] = $1/\lambda^2$
Lack of memory: P(X>s+t|X>s) = P(X>t)

Standard Normal Distribution

Z ~ N(0,1)
PMF: ø(z) = $1/\sqrt{(2\pi)}*e^{-1/2*z\wedge 2}$
CDF: Φ(z) = P(Z≤z) = ∫ø(t)dt
Φ(0) = 0.5
P(Z≤-z) = P(Z≥z)
Φ(-z) = 1-Φ(z)
P(a≤Z≤b) = Φ(b)-Φ(a)
P(-a≤Z≤-b) = Φ(a)-Φ(b)

Linear Combination

Y ~ N(μ_Y, σ^2_Y)
E[Y] = Σc_iE[X_i]
V[Y] = Σc^2_iV[X_i$]^2$
X̄ = 1/nΣX_i
E[X̄] = μ
V[X̄] = σ^2/n

Y = c_1X_1+...+c_nX_n

Sample Covariance

cov = ((Σx_iy_i)-(Σx_i)(Σy_i)/n)/(n-1)

Line of Best Fit

y = a+Bx
B = ((Σx_iy_i)-(Σx_i)(Σy_i)/n)/((Σx^2_i)-(Σx_i$)^2$/n)