

### Math 215

#### Chapter 7 • Sampling Distributions

- Mean of  $\bar{x}$ :  $\mu$
- Standard deviation of  $\bar{x}$ :  $\sigma_x = \sigma/\sqrt{n}$
- $t$  value for  $\bar{x}$ :  $t = \frac{\bar{x} - \mu}{\sigma_x}$
- (Note: If  $n < 30$ , population must be normal, otherwise it doesn't matter)
- Population proportion:  $p = X/n$
- Sample proportion:  $\hat{p} = x/n$
- Mean of  $\hat{p}$ :  $\mu_{\hat{p}} = p$
- Standard deviation of  $\hat{p}$ :  $\sigma_{\hat{p}} = \sqrt{p(1-p)}$
- $z$  value for  $\hat{p}$ :  $z = \frac{\hat{p} - p}{\sigma_{\hat{p}}}$

(Note: Necessary conditions are  $np > 5$  and  $nq > 5$ )

#### Chapter 8 • Estimation of the Mean and Proportion

- Point estimate of  $\mu$ :  $\bar{x}$
- Confidence interval for  $\mu$  when  $\sigma$  is known:

$$\bar{x} \pm z_{\alpha/2} \text{ where } \sigma_x = \sigma/\sqrt{n}$$

(Note: If  $n < 30$ , population must be normal)

Confidence interval for  $\mu$  when  $\sigma$  is not known:

$$\bar{x} \pm t_{\alpha/2} \text{ where } \sigma_x = s/\sqrt{n} \text{ and } t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

(Note: If  $n < 30$ , population must be normal)

Margin of error of the estimate of  $\mu$ :

$$E = z_{\alpha/2} \sigma_x \text{ or } E = t_{\alpha/2} s_x$$

• Sample size to estimate  $\mu$ :  $n = z^2 \sigma_x^2 / E^2$

• Confidence interval for  $p$  for a large sample:

$$\hat{p} \pm z_{\alpha/2} \sigma_{\hat{p}} \text{ where } \sigma_{\hat{p}} = \sqrt{\hat{p}(1-\hat{p})}$$

• Margin of error of the estimate of  $p$ :

$$E = z_{\alpha/2} \sigma_{\hat{p}}$$

• Sample size to estimate  $p$ :  $n = z^2 p(1-p) / E^2$

#### Chapter 9 • Hypothesis Tests about the Mean and Proportion

• Critical Value Approach:

Step 1: State the null and alternative hypotheses.

Step 2: Determine the rejection and non-rejection regions.

Step 3: Calculate the observed value of the test statistic.

Step 4: Make a decision and write a conclusion.

• Test of hypothesis about  $\mu$  when  $\sigma$  is known:

$$t_{\text{observed}} = \frac{\bar{x} - \mu_0}{\sigma_x}$$

(Note: If  $n < 30$ , population must be normal, otherwise it doesn't matter)

• Test of hypothesis about  $\mu$  when  $\sigma$  is not known:

$$t_{\text{observed}} = \frac{\bar{x} - \mu_0}{s_x}$$

(Note: If  $n < 30$ , population must be normal, otherwise it doesn't matter)

• Test of hypothesis about  $p$  for a large sample:

$$z_{\text{observed}} = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}}$$

where  $\sigma_{\hat{p}} = \sqrt{p_0(1-p_0)}$

#### Chapter 10 • Estimation and Hypothesis Testing: Two Populations

• Confidence interval for  $\mu_1 - \mu_2$  when  $\sigma_1$  and  $\sigma_2$  unknown but equal:

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where  $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

• Test of hypothesis about  $\mu_1 - \mu_2$  when  $\sigma_1$  and  $\sigma_2$  unknown but equal:

$$t_{\text{observed}} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where  $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

• Confidence interval for  $\mu_1$  in paired or matched samples:

$$\bar{d} \pm t_{\alpha/2} s_d$$

where  $\bar{d} = \frac{\sum d}{n}$  and  $s_d = \sqrt{\frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1}}$

• Test of hypothesis about  $\mu_1$  in paired or matched samples:

$$t_{\text{observed}} = \frac{\bar{d} - \mu_0}{s_d}$$

where  $\bar{d} = \frac{\sum d}{n}$  and  $s_d = \sqrt{\frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1}}$

• Confidence interval for  $\mu_1 - \mu_2$ :

$$(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sigma_{\bar{x}_1 - \bar{x}_2}$$

where  $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$  and  $\sigma_1 = \frac{s_1}{\sqrt{n_1}}$ ,  $\sigma_2 = \frac{s_2}{\sqrt{n_2}}$

• Test of hypothesis about  $\mu_1 - \mu_2$ :

$$z_{\text{observed}} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}}$$

where  $\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

and  $\sigma_1 = \frac{s_1}{\sqrt{n_1}}$  or  $\sigma_1 = \frac{\sigma_1}{\sqrt{n_1}}$ ,  $\sigma_2 = \frac{s_2}{\sqrt{n_2}}$  or  $\sigma_2 = \frac{\sigma_2}{\sqrt{n_2}}$

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#### Chapter 11 • Chi-Square Tests

• Goodness of fit test:

$H_0$ : The proportions (percentages) in the categories follow the distribution hypothesized.

$H_a$ : The proportions (percentages) in the categories do not follow the distribution hypothesized.

$$\chi^2_{\text{observed}} = \sum \frac{(O - E)^2}{E}$$

Expected frequency of a category:  $E = np$

Degrees of freedom:  $df = k - 1$  where  $k =$  number of categories

#### Contingency Tables - Test of Independence

$H_0$ : The row and column variables of contingency table are independent (i.e. not related).

$H_a$ : The row and column variables of contingency table are NOT independent (i.e. are related).

$$\chi^2_{\text{observed}} = \sum \frac{(O - E)^2}{E}$$

Expected frequency of cell:  $E = \frac{(\text{Row total})(\text{Column total})}{\text{Sample size}}$

Degrees of freedom:  $df = (R - 1)(C - 1)$  where  $R =$  # of rows (categories) and  $C =$  # of columns (categories) in contingency table

#### Contingency Tables - Test of Homogeneity

$H_0$ : The proportions of elements that belong to different categories are the same in two or more different populations.

$H_a$ : The proportions of elements that belong to different categories are NOT the same in two or more different populations.

(Note: Calculations and degrees of freedom same as for test of independence)

• Confidence interval for population variance  $\sigma^2$ :

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

where  $s^2 = \frac{\sum (x - \bar{x})^2}{n-1}$

(Note: Confidence interval for population standard deviation is found by taking square roots of confidence interval for population variance)

• Test of hypothesis about  $\sigma^2$ :

$$F_{\text{observed}} = \frac{s^2}{\sigma_0^2}$$

$H_0: \sigma^2 < \sigma_0^2$  OR  $H_0: \sigma^2 > \sigma_0^2$  OR  $H_0: \sigma^2 = \sigma_0^2$

$H_a: \sigma^2 > \sigma_0^2$  OR  $H_a: \sigma^2 < \sigma_0^2$  OR  $H_a: \sigma^2 \neq \sigma_0^2$

$t_{\text{observed}} = \frac{s^2 - \sigma_0^2}{\sigma_0^2}$

where  $MEP = z_{\alpha/2}(\sigma_0) \text{ and } MCFP = z_{\alpha/2}(\sigma_0)$

$df(\text{numerator}) = k - 1$  and  $df(\text{denominator}) = n - k$

$$SSR = \sum \left( \frac{x_i^2}{n_i} - \frac{(\sum x_i)^2}{n} \right)$$

$$SSW = \sum \left( \frac{x_i^2}{n_i} - \frac{(\sum x_i)^2}{n} \right)$$

$k =$  the number of different samples or treatments

$n =$  the size of sample  $i$

$N =$  the size of all the values in sample

$\Sigma x =$  the sum of values in all samples  $= \sum x_i = \sum x_1 + \sum x_2 + \dots + \sum x_k$

$\Sigma x^2 =$  the sum of squares of all the values in all the samples

#### Chapter 13 • Simple Linear Regression

• Simple linear regression model:  $y = a + bx + e$

• Estimated regression model:  $\hat{y} = a + bx$

where  $b = \frac{SS_{xy}}{SS_x}$  and  $a = \bar{y} - b(\bar{x} - \bar{x}) = \bar{y} - b\bar{x}$

$$SS_{xy} = \sum (x - \bar{x})(y - \bar{y})$$

$$SS_x = \sum (x - \bar{x})^2 \text{ and } SS_y = \sum (y - \bar{y})^2$$

• Confidence interval for  $\beta$ :

$$\hat{\beta} \pm t_{\alpha/2} s_{\hat{\beta}}$$

where  $s_{\hat{\beta}} = \sqrt{\frac{MSE}{\sum (x_i - \bar{x})^2}}$  and  $MSE = \frac{SS_{res}}{n-2}$

(Note:  $t$  distribution has  $n-2$  degrees of freedom)

• Test of hypothesis about  $\beta$ :

$$t_{\text{observed}} = \frac{\hat{\beta} - \beta_0}{s_{\hat{\beta}}}$$

$H_0: \beta > 0$  OR  $H_0: \beta < 0$  OR  $H_0: \beta = 0$

$H_a: \beta > 0$  OR  $H_a: \beta < 0$  OR  $H_a: \beta \neq 0$

$t_{\text{observed}} = \frac{\hat{\beta} - \beta_0}{s_{\hat{\beta}}}$

• Confidence interval for  $\mu_{y|x}$ :

$$\hat{y} \pm t_{\alpha/2} s_{\hat{y}}$$

where  $s_{\hat{y}} = \sqrt{\frac{MSE}{n} \left( 1 + \frac{(x - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right)}$  and  $df = n - 2$

• Predictions for  $y$ :

$$\hat{y} \pm z_{\alpha/2} s_{\hat{y}}$$

where  $s_{\hat{y}} = \sqrt{\frac{MSE}{n} \left( 1 + \frac{(x - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right)}$  and  $df = n - 2$

• Linear correlation coefficient:

$$r = \frac{SS_{xy}}{\sqrt{SS_x SS_y}}$$

• Test of hypothesis about  $\rho$ :

$$t_{\text{observed}} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

$H_0: \rho > 0$  OR  $H_0: \rho < 0$  OR  $H_0: \rho = 0$

$H_a: \rho > 0$  OR  $H_a: \rho < 0$  OR  $H_a: \rho \neq 0$

$t_{\text{observed}} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$

• Coefficient of determination:

$$r^2 = \frac{SS_{reg}}{SS_{tot}}$$

$r^2 =$  proportion of variation in  $y$  variable that is explained by its linear relationship with the variable

