

### Basis

A set S is a basis for V if

1. S spans V
2. S is LI.

If S is a basis for V then every vector in V can be written in one and only one way as a linear combo of vectors in S and every set containing more than n vectors is LD.

### Basis Test

1. If S is a LI set of vectors in V, then S is a basis for V
2. If S spans V, then S is a basis for V

### Change of Basis

$$P[x]_{B'} = [x]_B$$

$$[x]_{B'} = P^{-1} [x]_B$$

$$[B \ B'] \rightarrow [I \ P^{-1}]$$

$$[B' \ B] \rightarrow [I \ P]$$

### Cross Product

$$\text{if } u = u_1i + u_2j + u_3k$$

AND

$$v = v_1i + v_2j + v_3k$$

THEN

$$u \times v = (u_2v_3 - u_3v_2)i - (u_1v_3 - u_3v_1)j + (u_1v_2 - u_2v_1)k$$

### Definition of a Vector Space

$u + v$  is within V

$$u+v = v+u$$

$$u+(v+w) = (u+v)+w$$

$$u+0 = u$$

$$u-u = 0$$

$cu$  is within V

$$c(u+v) = cu+cv$$

$$(c+d)u = cu+du$$

$$c(du) = (cd)u$$

$$1 \cdot u = u$$

### Diagonalizable Matrices

A is diagonalizable when A is similar to a diagonal matrix.

That is, A is diagonalizable when there exists an invertible matrix P such that  $P^{-1}AP$  is a diagonal matrix

### Dot Products Etc.

$$\text{length/norm } ||v|| = \sqrt{v_1^2 + \dots + v_n^2}$$

$$||cv|| = |c| ||v||$$

$$v / ||v|| \text{ is the unit vector}$$

$$\text{distance } d(u,v) = ||u-v||$$

$$\text{Dot product } u \cdot v = (u_1v_1 + \dots + u_nv_n)$$

$$n \cos(\theta) = u \cdot v / (||u|| ||v||)$$

$u$  &  $v$  are orthogonal when  $\text{dot}(u,v) = 0$

### Eigenstuff

The scalar  $\lambda(Y)$  is called an **Eigenvalue** of A when there is a nonzero vector x such that  $Ax = Yx$ .

Vector x is an **Eigenvector** of A corresponding to  $\lambda$ .

The set of all eigenvectors with the zero vector is a subspace of  $\mathbb{R}^n$  called the **Eigenspace** of  $\lambda$ .

1. Find Eigenvalues:  $\det(YI - A) = 0$

2. Find Eigenvectors:  $(YI - A)x = 0$

If A is a triangular matrix then its eigenvalues are on its main diagonal

### Gram-Schmidt Orthonormalization

$$1. B = \{v_1, v_2, \dots, v_n\}$$

$$2. B' = \{w_1, w_2, \dots, w_n\}:$$

$$w_1 = v_1$$

$$w_2 = v_2 - \text{proj}_{w_1} v_2$$

$$w_3 = v_3 - \text{proj}_{w_1} v_3 - \text{proj}_{w_2} v_3$$

$$w_n = v_n - \dots$$

$$3. B'' = \{u_1, u_2, \dots, u_n\}:$$

$$u_i = w_i / ||w_i||$$

$B''$  is an orthonormal basis for V

$$\text{span}(B) = \text{span}(B'')$$

### Important Vector Spaces

$$\mathbb{R}^n$$

$$C(-\infty, +\infty)$$

$$C[a, b]$$

$$P$$

$$P_n$$

$$M_{m,n}$$

### Inner Products

$$||u|| = \sqrt{\langle u, u \rangle}$$

$$d(u,v) = ||u-v||$$

$$\cos(\theta) = \langle u, v \rangle / (||u|| ||v||)$$

$u$  &  $v$  are orthogonal when  $\langle u, v \rangle = 0$

$$\text{proj}_v u = \langle u, v \rangle / \langle v, v \rangle \cdot v$$

### Kernal

For  $T: V \rightarrow W$  The set of all vectors v in V that satisfies

$T(v)=0$  is the kernal of T.  $\ker(T)$  is a subspace of v.

For  $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$  by  $T(x)=Ax$   $\ker(T)$  = solution space of  $Ax=0$  &  $Cspace(A) = \text{range}(T)$

### Linear Combo

v is a linear combo of  $u_1 \dots u_n$

.

### Linear Independence

a set of vectors S is LI if  $c_1v_1 + \dots + c_kv_k = 0$  has only the trivial solution.

If there are other solutions S is LD. A set S is LI iff one of its vectors can be written as a combo of other S vectors.

### Linear Transformation

V & W are Vspaces.  $T: V \rightarrow W$  is a linear transformation of V into W if:

$$1. T(u+v) = T(u) + T(v)$$

$$2. T(cu) = cT(u)$$

### Non-Homogeny

If  $x_p$  is a solution to  $Ax = b$  then every solution to the system can be written as  $x = x_p$

### Nullity

$$\text{Nullspace}(A) = \{x \in \mathbb{R}^n : Ax = 0\}$$

$$\text{Nullity}(A) = \dim(\text{Nullspace}(A))$$

$$= n - \text{rank}(A)$$

### Orthogonal Sets

Set S in V is orthogonal when every pair of vectors in S is orthogonal. If each vector is a unit vector, then S is orthonormal

### One-to-One and Onto

T is one-to-one **iff**  $\ker(T) = \{0\}$

T is onto **iff**  $\text{rank}(T) = \dim(W)$

If  $\dim(T) = \dim(W)$  then T is one-to-one **iff** it is onto

### Rank and Nullity of T

$\text{nullity}(T) = \dim(\text{kernal})$

$\text{rank}(T) = \dim(\text{range})$

$\text{range}(T) + \text{nullity}(T) = n$  (in  $m \times n$ )

$\dim(\text{domain}) = \dim(\text{range}) + \dim(\text{kernal})$

### Rank of a Matrix

$\text{Rank}(A) = \dim(\text{Rspace}) = \dim(\text{Cspace})$

### Similar Matrices

For square matrices A and A' of order n, A' is similar to A when there exists an invertible matrix P such that  $A' = P^{-1}AP$

### Spanning Sets

$S = \{v_1 \dots v_k\}$  is a subset of vector space V. S spans V if every vector in v can be written as a linear combo of vectors in S.

### Test for Subspace

1.  $u+v$  are in W
2.  $cu$  is in w



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