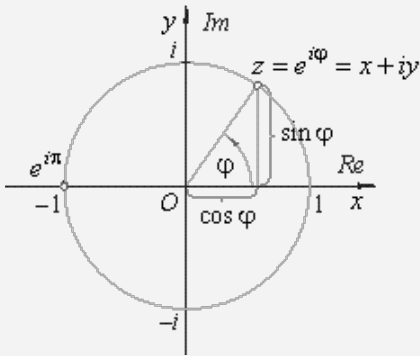


Cheatography

DIP Exam 2 Cheat Sheet

by Sawyer McLane (samclane) via cheatography.com/32204/cs/9879/

Complex Unit Circle



Discrete Fourier Transform

$$X_k \stackrel{\text{def}}{=} \sum_{n=0}^{N-1} x_n \cdot e^{-2\pi i k n / N}, \quad k \in \mathbb{Z}$$

Sinc Definition

$$\text{sinc}(x) = \begin{cases} 1 & \text{for } x = 0 \\ \frac{\sin x}{x} & \text{otherwise,} \end{cases}$$

2D DFT Definition

$$F[k, l] = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f[m, n] e^{-j2\pi \left(\frac{k}{M} m + \frac{l}{N} n \right)}$$

2D Continuous Fourier Transform

$$F(u, v) = \int \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(u x + v y)} dx dy$$

Wrap Around Error

Solved by zero padding

If $f(x)$ and $h(x)$ are A and B samples respectively, pad $f(x)$ and $h(x)$ with zeros so both have length $P \geq A+B-1$

If not zero, creates discontinuity called "frequency leakage", equivalent to convolving with $\text{sinc}()$ function

Reduced by multiplying with function that tapers smoothly to zero (windowing or apodizing)

Butterworth Lowpass Filter

$$H(u, v) = \frac{1}{1 + \left(\frac{D(u, v)}{D_0} \right)^{2n}}$$

D_0 is cutoff freq and $D(u, v)$ is distribution of (u, v) from centered origin. n is order

DFT Table

2D Convolution

$$z[m, n] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k, l] g[m-k, n-l]$$

Spatial Shift Theorem

$$\mathcal{F}\{f(t-t_0)\}(s) = e^{-j2\pi s t_0} F(s)$$

Spatial transform only affects FT phase

Conjugate Symmetry

$F^*(u, v) = F(-u, -v)$ (Conjugate Symmetry)

$F^*(-u, -v) = -F(u, v)$ (Conjugate Asymmetry)

Fourier Spectrum and Phase Angle

$$\begin{aligned} F(u, v) &= |F(u, v)| e^{j\phi(u, v)} \\ |F(u, v)| &= |F^*(u, v)| = |F^*(u, v)|^2 \\ \phi(u, v) &= \tan^{-1} \left[\frac{I(u, v)}{R(u, v)} \right] \end{aligned}$$

Steps for Filtering

- 1 + 2. Given $f(x, y)$ is $M \times N$, zero pad to $2M \times 2N$ ($P \times Q$)
3. Multiply by $(-1)^{x+y}$ to center
4. Take DFT of $f(x, y)$ to get $F(u, v)$
5. Generate symmetric filter $H(u, v)$ of size $P \times Q$
6. Get processed image $g_p(x, y) = \{\text{real}[F^{-1}\{G(u, v)\}] * (-1)^{x+y}\}$

Laplacian in Freq. Domain

$$\begin{aligned} H(u, v) &= -4v^2(u^2 + v^2) \\ H(u, v) &= -4v^2 \left[\left(\frac{u}{2} \right)^2 + \left(\frac{v}{2} \right)^2 \right] \\ H(u, v) &= -4v^2 D^2 + 4u^2 v \\ \nabla^2 f(x, y) &= \mathcal{F}^{-1} \{ H(u, v) F(u, v) \} \end{aligned}$$

Enhancement: $g(x, y) = f(x, y) + c \nabla^2 f(x, y)$, $c = -1$
 $g(x, y) = \mathcal{F}^{-1} \{ (F(u, v) H(u, v) - F(u, v)) \}$
 $g(x, y) = \mathcal{F}^{-1} \{ (1 - H(u, v)) F(u, v) \}$
 $g(x, y) = \mathcal{F}^{-1} \{ (1 + 4v^2 D^2) F(u, v) \}$

Impulse Train Definition

$$\psi_P(t) \stackrel{\text{def}}{=} \sum_{n=-\infty}^{\infty} \delta(t - nP)$$

Convolution Definition

$$\begin{aligned} (f * g)(n) &= \sum_{m=-\infty}^{\infty} f(m) g[n - m] \\ &= \sum_{m=-\infty}^{\infty} f[n - m] g(m) \end{aligned}$$

2D Sampling

$$\sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(x - nX) \delta(y - mY)$$

Frequency Shift Theorem

$$\mathcal{F}\{e^{-j2\pi f_0 t}\} = F(f - f_0)$$

DC Component

$$F(0, 0) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n) = MN \bar{f}(x, y)$$

Gaussian Filter

$$\begin{aligned} \text{Low-Pass: } H(u) &= A e^{-\frac{u^2}{2\sigma^2}} \\ h(x) &= \sqrt{2\pi\sigma} A e^{-\frac{x^2}{2\sigma^2}} \\ \text{High-Pass (wide narrow band): } H(u) &= A e^{\frac{u^2}{2\sigma^2}} - B e^{\frac{u^2}{2\sigma^2}} \\ h(x) &= \sqrt{2\pi\sigma} A e^{-\frac{x^2}{2\sigma^2}} - \sqrt{2\pi\sigma} B e^{-\frac{x^2}{2\sigma^2}} \end{aligned}$$

Unsharp, Highboost, High-Emphasis

$$g(x, y) = \mathcal{F}^{-1} \{ (1 + k H_{HP}(u, v)) F(u, v) \}$$

$g_{\text{mask}}(x, y) = f(x, y) - \text{flp}(x, y)$

$g(x, y) = f(x, y) + k * g_{\text{mask}}(x, y)$

$k=1$, unsharp

$k > 1$, highboost

Table C.2 Fourier Transform Pairs		Table C.1 Fourier Transform Theorems	
$x(t)$	$X(f)$	Name	Transform Pair
1. $\delta(t)$	$\int_{-\infty}^{\infty} \delta(t) e^{-j2\pi ft} dt = 1$	1. Linearity	$aX(f) + bY(f) \leftrightarrow aX(f) + bY(f)$
2. $\delta(t - t_0)$	$e^{-j2\pi ft_0}$	2. Scale change	$X(at) \leftrightarrow \frac{1}{ a } X(f/a)$
3. $\text{sinc}(at)$	$\frac{1}{ a } \text{rect}(f/a)$	3. Time reversal	$X(-f) \leftrightarrow x(-t)$
4. $e^{-\alpha t }$	$\frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}$	4. Complex conjugation	$X^*(f) \leftrightarrow x^*(-t)$
5. $\cos(\omega_0 t)$	$\frac{1}{2} [\delta(f - \omega_0) + \delta(f + \omega_0)]$	5. Duality	$X(f) \leftrightarrow x(-t)$
6. $e^{-\alpha t } \cos(\omega_0 t)$	$\frac{\alpha}{\alpha^2 + 4\pi^2 f^2} [\delta(f - \omega_0) + \delta(f + \omega_0)]$	6. Time shift	$X(f - f_0) \leftrightarrow x(t) e^{-j2\pi f_0 t}$
7. $e^{-\alpha t } \sin(\omega_0 t)$	$\frac{\alpha}{\alpha^2 + 4\pi^2 f^2} [\delta(f - \omega_0) - \delta(f + \omega_0)]$	7. Frequency translation	$X(f - f_0) \leftrightarrow x(t) e^{j2\pi f_0 t}$
8. t	$j \delta'(f)$	8. Modulation	$X(f) \cos(2\pi f_0 t) \leftrightarrow \frac{1}{2} [X(f - f_0) + X(f + f_0)]$
9. t^2	$-j^2 \delta''(f)$	9. Time differentiation	$\frac{d^2 X(f)}{df^2} \leftrightarrow (2\pi j)^2 X(t)$
10. $\text{sinc}(t)$	$\text{rect}(f)$	10. Time integration	$\int_{-\infty}^{\infty} X(f) df \leftrightarrow X(0)$
11. $\cos(2\pi f_0 t)$	$\frac{1}{2} [\delta(f - f_0) + \delta(f + f_0)]$	11. Convolution	$X(f) Y(f) \leftrightarrow x(t) * y(t)$
12. $\cos(2\pi f_0 t) \cos(2\pi f_1 t)$	$\frac{1}{4} [\delta(f - f_0 - f_1) + \delta(f - f_0 + f_1) + \delta(f + f_0 - f_1) + \delta(f + f_0 + f_1)]$	12. Multiplication	$X(f) Y(f) \leftrightarrow X(f) * Y(f)$

Fourier Series Definition

$$F_x(x) = a_0 + \sum_{k=1}^{\infty} (a_k \cos(kx) + b_k \sin(kx))$$

Convolution Theorem

$$\begin{aligned} \mathcal{F}\{f * g\} &= \mathcal{F}\{f\} \cdot \mathcal{F}\{g\} \\ \mathcal{F}\{f \cdot g\} &= \mathcal{F}\{f\} * \mathcal{F}\{g\} \\ f * g &= \mathcal{F}^{-1}\{\mathcal{F}\{f\} \cdot \mathcal{F}\{g\}\} \\ f \cdot g &= \mathcal{F}^{-1}\{\mathcal{F}\{f\} * \mathcal{F}\{g\}\} \end{aligned}$$

Space convolution = frequency multiplication

Center DC

To shift **F(0,0)** (DC Component) to center, multiply by **(-1)^{x+y}**

Power Spectrum

$$P(u,v) = |F(u,v)|^2$$

Total power of image is just sum of P(u,v) over P-1,Q-1

$$a = 100[\text{doublesum } P(u,v)/Pt]$$



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