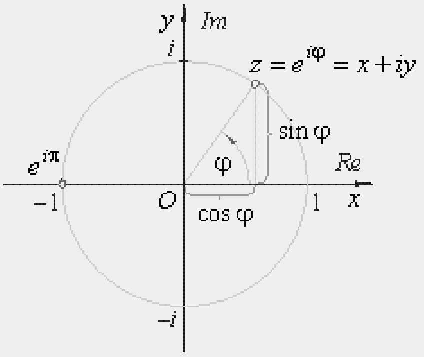
DIP Exam 2 Cheat Sheet

by samclane via cheatography.com/32204/cs/9879/

Complex Unit Circle



Discrete Fourier Transform

$$X_k \stackrel{\text{def}}{=} \sum_{n=0}^{N-1} x_n \cdot e^{-2\pi i k n/N}, \quad k \in \mathbb{Z}$$

Sinc Definition

$$\text{sinc}(x) = \begin{cases} 1 & \text{for } x = 0 \\ \frac{\sin x}{x} & \text{otherwise,} \end{cases}$$

2D DFT Definition

$$F[k,l] = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f[m,n] e^{-j2\pi\left(\frac{k}{M}m + \frac{l}{N}n\right)}$$

2D Continuous Fourier Transform

$$F(u,v) = \int\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx\, dy$$

Wrap Around Error

Solved by zero padding

If f(x) and h(x) are A and B samples respectively, pad f(x) and h(x) with zeros so both have length P>=A+B-1

If not zero, creates discontinuity called "frequency leakage", equivalent to convolving with sinc() function

Reduced by multiplying with function that tapers smoothly to zero (windowing or apodizing)

Butterworth Lowpass Filter

$$H(u,v) = \frac{1}{1 + [\frac{D(u,v)}{D_0}]^{2n}}$$

D0 is cutoff freq and D(u,v) is distribution of (u,v) from centered origin. n is order

DFT Table

Table C.2 Fourier Transform Pairs

	x(t)		X(f)
1.	$\Pi(t/\tau)$		$\tau\,\text{sinc}(\tau f)$
2.	$\Lambda(t/\tau)$		$\tau\,\text{sinc}^2(\tau f)$
3.	$\text{sinc}(at)$	$a > 0$	$\frac{1}{a}\Pi(f/a)$
4.	$e^{-at}u(t)$	$a > 0$	$\frac{1}{a + j2\pi f}$
5.	$te^{-at}u(t)$	$a > 0$	$\frac{1}{(a + j2\pi f)^2}$
6.	$e^{-a\vert t\vert}$	$a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
7.	e^{-at^2}		$\sqrt{\frac{\pi}{a}}e^{-(\pi f)^2/a}$
8.	1		$\delta(f)$
9.	$\delta(t)$		1
10.	$\text{sgn}(t)$		$1/j\pi f$
11.	$u(t)$		$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
12.	$\cos(2\pi f_0 t + \theta)$		$\frac{1}{2}e^{j\theta}\delta(f - f_0) + \frac{1}{2}e^{-j\theta}\delta(f + f_0)$
13.	$\sum_{n=-\infty}^{\infty} \delta(t - nT_0)$		$f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s)$, $f_s = \frac{1}{T_0}$

Table C.1 Fourier Transform Theorems

	Name	Transform Pair
1.	Linearity	$ax(t) + by(t) \leftrightarrow aX(f) + bY(f)$
2.	Scale change	$x(at) \leftrightarrow \frac{1}{\vert a\vert}X(f/a)$
3.	Time reversal	$x(-t) \leftrightarrow X(-f)$
4.	Complex conjugation	$x^*(t) \leftrightarrow X^*(-f)$
5.	Duality	$X(t) \leftrightarrow x(-f)$
6.	Time shift	$x(t - t_0) \leftrightarrow X(f)e^{-j2\pi f t_0}$
7.	Frequency translation	$x(t)e^{j2\pi f_0 t} \leftrightarrow X(f - f_0)$
8.	Modulation	$x(t)\cos 2\pi f_0 t \leftrightarrow \frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
9.	Time differentiation	$\frac{d^n x(t)}{dt^n} \leftrightarrow (j2\pi f)^n X(f)$
10.	Time integration	$\int_{-\infty}^{t} x(\lambda)d\lambda \leftrightarrow (j2\pi f)^{-1}X(f) + \frac{1}{2}X(0)\delta(f)$
11.	Convolution	$x(t) * y(t) \leftrightarrow X(f)Y(f)$
12.	Multiplication	$x(t)y(t) \leftrightarrow X(f) * Y(f)$

Fourier Series Definition

$$F_n(x) = a_0 + \sum_{k=1}^{k=n} \left(a_k \cos(kx) + b_k \sin(kx)\right).$$

Convolution Theorem

$$\mathcal{F}\{f * g\} = \mathcal{F}\{f\} \cdot \mathcal{F}\{g\}$$
$$\mathcal{F}\{f \cdot g\} = \mathcal{F}\{f\} * \mathcal{F}\{g\}$$
$$f * g = \mathcal{F}^{-1}\{\mathcal{F}\{f\} \cdot \mathcal{F}\{g\}\}$$
$$f \cdot g = \mathcal{F}^{-1}\{\mathcal{F}\{f\} * \mathcal{F}\{g\}\}$$

Space convolution = frequency multiplication

2D Convolution

$$x[m,n] = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} x[i,j] \cdot \delta[m-i, n-j]$$

Spatial Shift Theorem

$$\mathcal{F}\{f(t - t_0)\}(s) = e^{-j2\pi s t_0} F(s)$$

Spatial transform only affects FT phase

Conjugate Symetry

F*(u,v) = F(-u, -v) (Conjugate Symmetry)
F*(-u,-v) = -F(u,v) (Conjugate Asymmetry)

Fourier Spectrum and Phase Angle

$$F(u,v) = |F(u,v)| e^{j\phi(u,v)}$$
$$|F(u,v)| = [R^2(u,v) + I^2(u,v)]^{1/2}$$
$$\phi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$

Steps for Filtering

1 + 2. Given f(x,y) is MxN, zero pad to 2Mx2N (PxQ)

3. Multiply by $(-1)^{x+y}$ to center

4. Take DFT of f(x,y) to get F(u,v)

5. Generate symmetric filter H(u,v) of size PxQ

6. Get processed image gp(x,y)={real[$\mathcal{F}^{-1}${G(u,v)}} * $(-1)^{x+y}$

Laplacian in Freq. Domain

$$H(u,v) = -4\pi^2(u^2 + v^2)$$
$$H(u,v) = -4\pi^2\left[\left(u - \frac{P}{2}\right)^2 + \left(v - \frac{Q}{2}\right)^2\right]$$
$$H(u,v) = -4\pi^2 D^2 * (u,v)$$
$$\rightarrow \nabla^2 f(x,y) = \Im^{-1}\{H(u,v)F(u,v)\}$$

$$\text{Enhancement: } g(x,y) = f(x,y) + c\nabla^2 f(x,y),\ c = -1$$
$$g(x,y) = \Im^{-1}\{F(u,v)H(u,v) - F(u,v)\}$$
$$g(x,y) = \Im^{-1}\{[1 - H(u,v)]F(u,v)\}$$
$$g(x,y) = \Im^{-1}\{[1 + 4\pi^2 D^2(u,v)]F(u,v)\}$$

Impulse Train Definition

$$\psi_P(t) \triangleq \sum_{m=-\infty}^{\infty} \delta(t - mP)$$

Convolution Definition

$$(f * g)[n] \stackrel{\text{def}}{=} \sum_{m=-\infty}^{\infty} f[m]\, g[n - m] = \sum_{m=-\infty}^{\infty} f[n - m]\, g[m].$$

2D Sampling

$$\sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \delta(x - nX)\delta(y - mY)$$

Frequency Shift Theorem

$$\mathcal{L}\left[e^{-at} f(t)\right] = F(s + a)$$

Center DC

To shift **F(0,0)** (DC Component) to center, multiply by **$(-1)^{x+y}$**

Power Spectrum

$$P(u,v) = |F(u,v)|^2$$

Total power of image is just sum of P(u,v) over P-1,Q-1

a = 100[doublesum P(u,v)/Pt]

DC Component

$$F(0,0) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) = MN\overline{f}(x,y)$$

Gaussian Filter

Low-Pass: $H(u) = Ae^{\frac{-u^2}{2\sigma^2}}$

$$h(x) = \sqrt{2\pi}\sigma Ae^{-2\pi^2\sigma^2x^2}$$

High-Pass (wide-narrow band): $H(u) = Ae^{\frac{-u^2}{2\sigma_1^2}} - Be^{\frac{-u^2}{2\sigma_2^2}}$

$$h(x) = \sqrt{2\pi}\sigma_1 Ae^{-2\pi^2\sigma_1^2x^2} - \sqrt{2\pi}\sigma_2 Be^{-2\pi^2\sigma_2^2x^2}$$

Unsharp, Highboost, High-Emphasis

$$g(x,y) = \mathfrak{F}^{-1}\{[1 + kH_{HP}(u,v)]\,F(u,v)\}$$

gmask(x,y) = f(x,y) - flp(x,y)
g(x,y) = f(x,y) + k*gmask(x,y)
k=1, unsharp
k>1, highboost