Quantitative Methods Final Exam Cheat Sheet

by rockcollector2 via cheatography.com/22080/cs/4782/

Statistics & Probability Notes

Standard Deviation

$$\sigma^2 = \frac{\sum_{i=1}^{n}(\mu - x_i)^2}{n}$$

Quartiles

IQR = Q3 = Q1

Outliers are beyond: Q1 - 1.5 * IQR, Q3 + 1.5 + IQR

Calc Entry for Basic Stats

1-VAR Stats

Correlation (R^2 near 1 is better fit)

$$\text{Correlation: } R = \frac{1}{n-1}\sum\left(\frac{x_i-\bar{x}}{\sigma_x}\right)\left(\frac{y_i-\bar{y}}{\sigma_y}\right)$$

>COMBINATORIAL PROBABILITY<

4-Digit PIN with repetition = 10^4

4-Digit PIN without repetition = 10!/6! = 5040

Permutations

How many ways can 6 people be ranked? Ranking n objects leads to n!

How many ways can 6 people be ranked into 3 places:

n*(n-1)*...*(n-r+1) = n!/(n-r)! or 6 nPr 3

Combinations

Combinations = choosing a certain number of objects from a given set (no order).

N choose R or N!/(N-R)!R! or nCr

Probability

P is a function on the space of events such that:

- $P(E)$ is always a number between 0 and 1
- $P(S) = 1$ (something always happens)
- $P(E_1 \cup E_2 \cup \ldots) = \sum P(E_1) + P(E_2) + \ldots$ whenever these events do not overlap.

Example

3 cities from 15 are chosen randomly for a visit. A: 4 cost $800, B: 5 cost $300, C: 6 cost $100. What is the probability that the tour will cost $1000 or less?

- All from C 6 nCr 3
- Two from C, one from B or A (6nCr2)(5nCr1) + (6nCr2)(4nCr1)
- One from C, two from B (6nCr1)*((5nCr2)
- None from C, three from B (5nCr3)

Compute and sum: 20+75+60+60+30=245; Divide by the total 15nCr3 = 455

.538 or 53.8%

>RANDOM VARIABLES<

Example of Random Variable

X is the number of heads in 10 flips of a coin. P(X=4)?

(10nCr4)/2^10 = .205

X is sum of two dice.

P(X=5) = 4/36

P(9<=X<=11) = P(X=9)+P(X=10)+P(X=11)= 4/36+3/36+2/36

Binomial Random Variables

Binomial Random Variable represents the number of successes in n trials with probability p of success. The probability of 4 successes in 10 trials (p=0.5) is (10 nCr4_/2^10 = 0.205

Calc: binompdf(10,.5,8) = (n,p,r) r is number of successes

Math: (n r) $p^{r(1-p)}$n-r

Binomial Random Variables

Binomial with n=15 and p = .4

Compute:

P(X=3)=binompdf(15,.4,3)

P(X<=3)=binomcdf(15,.4,3)

P(X<3)=binomcdf(15,.4,2)

P(X>3)=1-binomcdf(15,.4,3)

P(X>=3)=1-binomcdf(15,.4,2)

P(4<x<=8)=binomcdf(15,.4,8)-binomcdf(15,.4,4)

P(1.3<X<1.7)=0

Continuous & Normal Random Variable

For C RV, P(X=3) or any other value is zero.
Probability is area under curve.
Normal RV is bell curve.
Total area under bell is 1.
P(X<a) is the are under the curve up to x=a.
1 Std. Dev. P(-1<Z<1)= .683
2 Std. Dev. P(-2<Z<2)=.954
3 Std. Dev. P(-3<Z<3)=.997

Pure Numbers

X is a random normal variable with mean -3 and standard deviation 0.7
P(-4<X<-3) = normalcdf(-4,-3,-3,.7)
P(X>-2) = normalcdf(-2,1E99,-3,.7)
P(X<=-3.5)=normalcdf(-1e((,-3.5,-3,.7)
P(X=-3) =0
P(|X-(-3)|>.7)=P(X<-3.7)+P(X>-2.3) = normalcdf(-1E99,-3.7,-3,.7)+normalcdf(-2.3,1E99,-3,.7)
A car model gets 24 mpg on the car sticker. The maker knows that this is normally distributed with a std dev of 3 mpg. What is the proportion of cars that get less than 20 mpg? P(X,20)=normalcdf(-1E99,20,24,3)=0.91

Conditional Probability

P(A|B) is the probability of A given that B happened.
P(A|B) = P(AintersectB) /P(B)
If P(A|B) = P(A) then independent.
P(AintersectB) = P(A)P(B)
If mutually exclusive P(A|B)=0

Bayes's Theorem

Bayes's Theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

Central Limit Theorem

As n becomes large, the sample mean will be distributed according to the normal distribution with parameters u and standard deviation - std dev/sqrt n
*As n gets large, the spread in the sample mean distribution narrows. This means that the sample mean is more likely to be near the true mean.

>INFERENTIAL STATISTICS<

Z-test approximated by normal distribution. If sample size is large or variance is known.
T-score/test is used when:
- sample size is below 30
- population standard deviation is unknown (estimated from your sample data)
otherwise use z-score/test.
Generally use 95% confidence level.
Z-Stat represents how many std. deviations away from the mean the sample mean is.
Std. dev is std dev/sqrt n
Null hypotheses assumes that whatever you are trying to prove did not happen.
p-value of 0.03 means there is a 3% chance of finding a difference as large as or larger than the one in your study given the null hypothesis is true.
If 0.05 or less you typically do not accept the null hypothesis.
Type 1 error: rejecting the null hypothesis when true
Type 2 error: accepting the null hypothesis when false

Two Sample T Test

Compute $t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ and use say smaller of the sample sizes for df.
Or use 2SampleTTest.

Calculus Notes

Derivatives and Tangent Lines

The derivative of a function f at x is the slope of the tangent at x. If all of the slopes are assembled you get f'(x) or df/dx.
If we know f'(a) and f(a), the the tangent line at x=a is y=f'(a)(x-a)+f(a)
Slope is f'(a) and line passes through (a,f(a))
Approximate slope use: nDeriv

Computing Areas & Integrals

The definite integral of f from a to b is the area underneath the curve from a to b.
Where f is negative, the area contributed is a negative area.
Use fnInt

Fundamental Theorem of Calculus

$$\int_a^b f'(x)dx = f(b) - f(a)$$

Limits

A function f(x) converges to a limit L at x=a if, for any given error tolerance, we can specify a range of x such that for any x in that range, f(x) is near L, near being given the tolerance.

Basic Derivative Rules

- $[1]' = 0$
- $[x]' = 1$
- $[x^2]' = 2x$
- $[x^3]' = 3x^2$
- $[x^{5.732}]' = 5.732x^{4.732}$
- $[1/x]' = [x^{-1}]' = \frac{-1}{x^2}$
- $[1/x^2]' = [x^{-2}]' = \frac{-2}{x^3}$
- $[\sqrt{x}]' = [x^{1/2}]' = \frac{1}{2\sqrt{x}}$
- $[\frac{1}{\sqrt{x}}]' = [x^{-1/2}]' = (-1/2)x^{-3/2}$
- $[x^n]' = nx^{n-1}$

Basic Derivative Rules

- $[e^x]' = e^x$
- $[\ln(x)]' = \frac{1}{x}$
- $[b^x]' = \ln(b)b^x$
- $[\log_b(x)]' = \frac{1}{\ln(b)x}$
- $[\sin(x)]' = \cos(x)$
- $[\cos(x)]' = -\sin(x)$
- $[\tan(x)]' = \frac{1}{\cos^2(x)}$
- $[\arcsin(x)]' = \frac{1}{\sqrt{1-x^2}}$
- $[\arccos(x)]' = \frac{-1}{\sqrt{1-x^2}}$
- $[\arctan(x)]' = \frac{1}{1+x^2}$

Basic Derivative Rules

- $[af(x)]' = af'(x)$
- $[f(bx)]' = bf'(bx)$
- $[f(x) + c]' = f'(x)$
- $[f(x) + g(x)]' = f'(x) + g'(x)$
- Product Rule: $[f(x) * g(x)]' = f'(x)g(x) + g'(x)f(x)$
- $[\frac{1}{f(x)}]' = \frac{-f'(x)}{(f(x))^2}$
- Quotient Rule: $[\frac{f(x)}{g(x)}]' = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$
- Chain Rule: $[f(g(x))]' = f'(g(x))g'(x)$

Critical Points

Min: f goes from decreasing to increasing
f' goes from negative to positive.
Max: f goes from increasing to decreasing.
f' goes from positive to negative.
Flat: f continues to change in the same way.
f' does not change sign.
f" gives concavity.
Concave up means second derivative is positive which means first derivative is increasing
Concave down: f"<0, f' decreasing.

Max/Min Word Problems

10 meters of string. maximum area dimensions?
Perimeter: P(l,w) = 2l_2w P=10
Area: A9l,w)=lw
a(l)=l(5-1)
max is at l=2.5

Newton's Method

1. Pick a, initial guess.
2. Compute tangent line approximation: y = f(a)+f'(a)(x-a)
3. Solve y=0 and get x = (f'(a)a-f(a))/f'(a)
4. Use x for the next guess. Repeat.

>Integrals, Series<

$F(x) = \int f(x)dx$	$f(x)$	$\frac{df}{dx} = f'(x)$
$\frac{1}{n+1}x^{n+1} + C$	x^n	nx^{n-1}
$\ln(x) + C$	$1/x$	$-1/x^2$
$e^x + C$	e^x	e^x
$x\ln(x) - x + C$	$\ln(x)$	$\frac{1}{x}$
$-\cos(x) + C$	$\sin(x)$	$\cos(x)$
$\sin(x) + C$	$\cos(x)$	$-\sin(x)$
$aF(x) + C$	$af(x)$	$af'(x)$
$\frac{1}{b}F(bx) + C$	$f(bx)$	$bf'(bx)$
$F(x) + cx + C$	$f(x) + c$	$f'(x)$
$F(x) + G(x) + C$	$f(x) + g(x)$	$f'(x) + g'(x)$

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Integration by Parts

This undoes the Product Rule:

$$\int f(u)g(u)du = f(x)G(x) - \int f'(u)G(u)du$$

Integration by Parts

- $\int u\cos(u)du$.
  1. $f(x) = x$, $f'(x) = 1$
  2. $g(x) = \cos(x)$ and $G(x) = \sin(x)$
  3. IBP: $\int u\cos(u)du = x\sin(x) - \int \sin(u)du$
  4. Simplify: $\int u\cos(u)du = x\sin(x) + \cos(x) + C$
  5. Check: $[x\sin(x) + \cos(x) + C]' = x\cos(x)$

Integration by Parts

This undoes the chain rule:

$$\int_a^b f(g(x))g'(x)dx = F(g(b)) - F(g(a))$$

Integration by Parts

Trick: $u = g(x)$, $du = g'(x)dx$ to get
$\int f(g(x))g'(x)dx = \int f(u)du = F(u) + C = F(g(x)) + C$

Arc Length

Pick points on curve and sum the distances: $\sum_i \sqrt{(\Delta x)_i^2 + (\Delta f)_i^2}$

Taylor Polynomials

- $f(x) \approx a + bx + cx^2 + dx^3 + \cdots$, $f(0) = a$
- $f'(x) \approx b + 2cx + 3dx^2 + \cdots$, $f'(0) = b$
- $f''(x) \approx 2c + 6dx + \cdots$, $f''(0) = 2c$
- $f'''(x) \approx 6d + \cdots$, $f'''(0) = 6d$

$f(x) \approx f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{6}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$
Error: $|f(x) - T_n(x)| \le \frac{x^{n+1}}{(n+1)!}M$ where $M \ge |f^{(n+1)}(x)|$

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