

Semantics

$\neg, \exists x, \forall y$ (Highest, do first)

 \wedge

V

→ (Lowest, do last)

Basic Equivalences	Proof Rules	Proof Rules (cont)	CS 310 Lecture 5 (cont)
Negation $\neg\neg A$	Conjunction (Conj) $A, B / A \wedge B$	Contradiction (Contr) $A, \neg A / \text{False}$	Terms to understand the concept of Array. o Element – Each item stored in an array is called an element.
Basic Equivalences	Simplification (Simp) $A \wedge B / A$ $A \wedge B / B$	Indirect Proof (IP) From $\neg A$, derive False / A	o Index – Each location of an element in an array has a numerical index.
Some Conversions $A \rightarrow B \equiv \neg A \vee B$ $\neg(A \rightarrow B) \equiv A \wedge \neg B$ $A \rightarrow B \equiv A \wedge \neg B \rightarrow \text{False}$	Addition (Add) A / \square $\square \vee B$ and B / \square $\square \vee B$	These are all fractions with the first term appearing on top and the second one on the bottom. / this slash denotes where a fraction will be located	
\wedge and \vee are associative $(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$ $(A \vee B) \vee C \equiv A \vee (B \vee C)$	Disjunctive Syllogism (DS) $A \vee \square$ $\square, \neg A / B$ $A \vee \square$ $\square, \neg A / B$		
\wedge and \vee are commutativity $A \wedge B \equiv B \wedge A$ $A \vee B \equiv B \vee A$	Modus Ponens (MP) $A, A \rightarrow B / B$	CS 310 Lecture 5 Array Data Structure A linear data structure defined as a collection of elements with the same or different data types.	CS 310 Lecture 5 Array Update Operation 1. Start 2. Set LA[K-1] = ITEM 3. Stop
\wedge and \vee are Distributivity $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$ $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$	Conditional Proof (CP) From A , derive \square $\square / A \rightarrow B$		CS 310 Lecture 5 Common Features of Linked List
	Double Negation (DN) $\neg\neg A / A$ $A / \neg\neg A$	They exist in both single and multiple dimensions	

CS 310 Lecture 5 (cont)

>Node: Each element in a LL is represented by a node, contains two components:

>Data: The actual data or value associated with the element.

>Next: Pointer: A reference or pointer to the next node in the LL.

>Head: The first node in a LL is called the "head." It serves as the starting point.

>Tail: The last node in a linked list is called the "tail."

>Data structures can be added to or removed from the LL during execution.

>Unlike an array, LL is a dynamically allocated DS that can grow and shrink.

>No elements need to be shifted after insertion and deletion.

CS 310 Lecture 5 (cont)

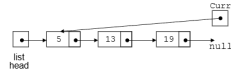
>Various DSs can be implemented using an LL, such as stack, queue, graphs, hash, etc.

>Linked list contains 0 or more nodes. Last node points to null(address 0)

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Linked List: Traversing all Nodes

>Visit each node in a LL: display contents, validate data, etc.



LIST-Traversal (L)
1. Curr = L.head
2. While Curr.next != NULL
3. PRINT Curr

LIST-Traversal (L)

1. Curr = L.head
2. While Curr.next != NULL
3. PRINT Curr

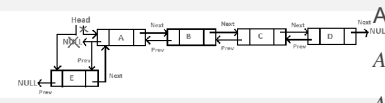
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Linked List: Searching a Node

LIST-Searching (L,k)

1. Curr = L.head
2. While Curr != NULL and Curr.key != k
3. Curr = Curr.next
4. return Curr

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Doubly-linked list: Inserting at the beginning

>The task can be performed by using the following 5 steps:

>Firstly, allocate a new node.

>Now put the required data in the new node.

>Make the next of new_node point to the current head of the DLL.

>Make the previous of the current head point to new_node.

>Lastly, point head to new_node.

Basic Equivalences

Disjunction

$A \vee \text{True} \equiv \text{True}$

$A \vee \text{False} \equiv A$

$A \vee A \equiv A$

$A \vee \neg A \equiv \text{True}$

Basic Equivalences

Absorption Laws

$A \wedge (A \vee B) \equiv A$

$A \vee (A \wedge B) \equiv A$

$A \wedge (\neg A \vee B) \equiv A \wedge B$

$A \vee (\neg A \wedge B) \equiv A \vee B$

Program Correctness

AA $\{Q(x/t)\} x := t$

(Assign- $\{Q\}$

ment axiom)

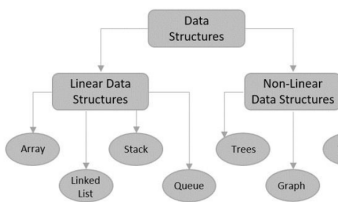
Conseq $P \rightarrow R$ $\{P\} S$
uence and $\{R\}$ $\{T\}$ and \square
rules (A $S \{Q\}$ $\square \rightarrow Q /$
& B) $/ \{P\} \square$ $\{P\} S$
 $\square \{Q\}$ $\{Q\}$

Loop invariants: A loop invariant is a condition that does not change after a loop has executed i.e. P

Derived Proof Rules

Modus Tollens (MT)	$A \rightarrow B, \neg B / \neg A$
Hypothetical Syllogism (HS)	$A \rightarrow B, B \rightarrow C / A \rightarrow C$
Proof by Cases (Cases)	$AVB, A \rightarrow C, \square / \square \rightarrow C / C$
Constructive Dilemma (CD)	$A \vee B, A \rightarrow C, B \rightarrow D / C \vee D$
Destructive Dilemma (DD)	$A \rightarrow B, C \rightarrow D, \neg B \vee \neg D / \neg A \vee \neg C$

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Operations in Arrays

- o Traverse – print all the array elements one by one.
- o Insertion – Adds an element at the given index.
- o Deletion – Deletes an element at the given index.
- o Search – Searches an element using the index or value.
- o Update – Updates an element at the given index.
- o Display – Displays the contents of the array.

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Array Search Operation

1. Start
2. Set J = 0
3. Repeat steps 4 and 5 while J < N
4. IF LA[J] == ITEM THEN GOTO STEP 6
5. Set J = J + 1
6. PRINT J, ITEM
7. Stop

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>Singly Linked List: Every node stores the address of the next node in the list and the last node has the next address NULL.

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Linked List: Operations

>Accessing Elements/Traversing: Accessing a specific element in a linked list takes O(n) time since nodes are stored in non-contiguous locations, so random access is not possible.

>Searching: Searching a node in an LL takes O(n) time, as the whole list needs to be traversed in the worst case.

>Insertion: If we are at the position where we insert the element, insertion takes O(1) time.

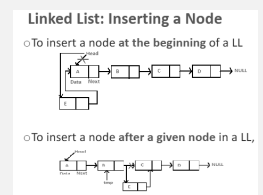
>Deletion a/Destroy the list: Deletion takes O(1) time if we know the element's position to be deleted.

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>Doubly Linked Lists: Each node has two pointers: one pointing to the next node and one pointing to the previous node. Allows for efficient traversal in both directions.

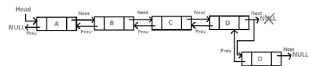
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LIST-Insert (L,x,k)

1. if L.Head == NULL
2. L.Head = x and Exit
3. While Curr.key !=k and Curr !=NULL
4. prevN = Curr
5. Curr = Curr.Next
6. If PrevN == NULL
7. x.next = L.Head
8. Head = x and exit
9. PrevN = Curr and Curr = Curr.Next
10. x.Next = Curr
11. PrevN.Next = x

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Doubly-linked list: Inserting at the end

> This can be done using the following 7 steps:

> Create a new node (say new_node).

> Put the value in the new node.

> Make the next pointer of new_node as null.

> If the list is empty, make new_node as the head.

> Otherwise, travel to the end of the linked list.

> Now make the next pointer of last node point to new_node.

> Change the previous pointer of new_node to the last node of the list.

Basic Equivalences

Absorption Laws

$$A \wedge (A \vee B) \equiv A$$

$$A \vee (A \wedge B) \equiv A$$

$$A \wedge (\neg A \vee B) \equiv A \wedge B$$

$$A \vee (\neg A \wedge B) \equiv A \vee B$$

Program Correctness

$$AA \quad \{Q(x/t)\} x := t$$

(Assignment axiom)

$$\begin{array}{ll} \text{Consequence} & P \rightarrow R \quad \{P\} S \\ & \text{and } \{R\} \quad \{T\} \text{ and } \Box \\ \text{rules (A \& B)} & S \{Q\} \quad \Box \rightarrow Q / \\ & / \{P\} \Box \quad \{P\} S \\ & \Box \{Q\} \quad \{Q\} \end{array}$$

Program Correctness (cont)

Composition rule $\{P\} S1 \{Q\}$ and $\{Q\} S2 \{R\} / \{P\} \Box \Box 1; S2 \{R\}$

If-then Rule $\{P \wedge C\} S \{P\}$ if $\{Q\}$ and $\Box C$ then Q S $\{Q\}$

Program Correctness (cont)

If-then-else rule $\{P \wedge C\} S1 \{Q\}$ and $\{P \wedge \neg C\} S2 \{Q\} / \{P\}$ if C then $S1$ else $\Box 2 \{Q\}$

While rule $\{P \wedge C\} S \{P\} / \{P\}$ while C do $S \{P \wedge \neg C\}$

Loop invariants: A loop invariant is a condition that does not change after a loop has executed i.e. P

Basic Equivalences

Conjunction

$$A \wedge \text{True} \equiv A$$

$$A \wedge \text{False} \equiv \text{False}$$

$$A \wedge A \equiv A$$

$$A \wedge \neg A \equiv \text{False}$$

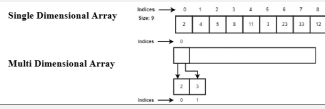
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Array Deletion Operation

1. Start
2. Set $J = K-1$
3. Repeat steps 4 and 5 while $J < N$
4. Set $LA[J] = LA[J + 1]$
5. Set $J = J+1$
6. Set $N = N-1$
7. Stop

N - is the size of the array

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Array Insertion Operation

1. Start
2. Create an Array of a desired datatype and size.
3. Initialize a variable 'i' as 0.
4. Enter the element at the i-th index of the array.
5. Increment i by 1
6. Repeat Steps 4 & 5 until the end of the array.

CS 310 Lecture 5 (cont)

7. Stop

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>Circular Linked Lists: A circular linked list is a type of linked list in which the first and the last nodes are also connected to form a circle. There is no NULL at the end.

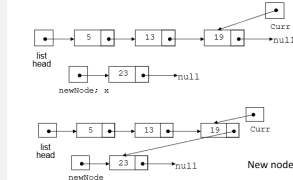
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Linked List: Empty List
 >If a list currently contains 0 nodes, it is called the empty list.
 >In this case, the list head points to null

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Linked List: Appending a Node

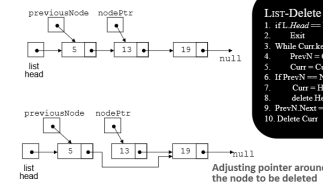


LIST-Append (L; x)
 1. if $L.head == NULL$
 2. $L.head = x$ and Exit
 3. $Curr = L.head$
 3. While $Curr.next != NULL$
 $Curr = Curr.next$
 4. $Curr.next = x$

New Node is added to the end of the list

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Linked List: Deleting a Node

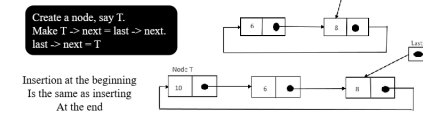


Adjusting pointer around the node to be deleted

LIST-Delete (L, k)
 1. if $L.Head == NULL$
 2. Exit
 3. While $Curr.key != k$ and $Curr != NULL$
 4. $PrevN = Curr$
 5. $Curr = Curr.Next$
 6. If $PrevN == NULL$
 7. $Curr = Head.Next$
 8. delete Head and exit
 9. $PrevN.Next = Curr.Next$
 10. Delete Curr

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Inserting at the beginning of the list



Circular-linked list operations:
 >Insertion: Inserting At the Beginning, at the end, and after a given node.
 >Deletion: Deleting from the Beginning, the end, and a Specific Node
 >Display: This process displays the elements of a CLL.

Basic Equivalences

Implication

$A \rightarrow True \equiv True$

$A \rightarrow False \equiv \neg A$

$True \rightarrow A \equiv A$

$False \rightarrow A \equiv True$

$A \rightarrow A \equiv True$

Basic Equivalences

De Morgan's Laws

$\neg(A \wedge B) \equiv \neg A \vee \neg B$

$\neg(A \vee B) \equiv \neg A \wedge \neg B$

Quantifiers

"An equivalence to be careful with"

$\exists x(p(x) \rightarrow q(x)) \equiv \forall x p(x) \rightarrow$

$\exists x q(x)$

Quantifiers

Negations of quantifiers

$\neg(\forall x W) \equiv \exists x \neg W$

$\neg(\exists x W) \equiv \forall x \neg W$

Quantifiers

Formalize English sentences
and entire arguments into FOPC

$\forall x$ quantifies a conditional

$\exists x$ quantifies a conjunction

$\forall x$ with conditional for “all,”
“every,” and “only.”

$\exists x$ with conjunction for “some,”
“there is,” and “not all.”

$\forall x$ with conditional or $\neg \exists x$ with conjunction for “no A is B.”

$\exists x$ with conjunction or $\neg \forall x$ with conditional for “not all A’s are B.”

Inference Rules FOPC

UI	Universal instantiation requires that t is free to replace x in $W(x)$:	$\forall x W(x) / \square$ $\square(t)$	There are two special cases for UI:	$\forall x W(x) / \square$ $\square(x)$	$\forall x W(x) / \square$ $\square(c)$
----	---	--	--	--	--

Inference Rules FOPC (cont)

EI	Existential generalization requires that t is free to replace x in $W(x)$	$W(t) /$ $\exists xW(x)$
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Inference Rules FOPC (cont)

EG	Existential generalization requires that \Box \Box is free to replace x in $W(x)$:	$W(t) /$ $\exists x W(x)$	There are two special cases for EG:
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Inference Rules FOPC (cont)

UG) / Universal	$\exists x W(x) /$
$\exists x W(x)$ Generalization	$W(c)$
<hr/>	
UI & EI Add A and E from	
problem; UG, EG, Take the	
away A and E in the problem.	

CS 310 Lecture 5

Array Traversal Operation

1. Start
2. Initialize an Array, LA. // 1.
Initialize an array called LA
3. Initialize, i = 0. // 2. Set i - 0
4. Print the LA[i] and increment i.
// 3. Repeat Steps 4-5 while i < N
5. Repeat Step 4 until the end of the array. // 4. Print LA[i]

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6. End // 5. Increment the value of i by one. (Set $i = i + 1$)

// represent possible modifications you can do that would still be counted as correct

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>An abstract data type (ADT) in data structure is a data type defined with the help of some attributes and some functions

>An abstract data type in the data structure can be

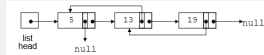
>A list data structure

>A stack data structure

>A queue data structure

>Unlike Arrays, Linked List elements are not stored at a contiguous location.

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Doubly-linked list: Operations

>Insertion: Inserting At the Beginning, at the end, after a given node, and before a given node.

>Deletion: Deleting from the Beginning, end, and a specific node of the list

>Display: This process displays the elements of a doubly LL.

CS 310 Lecture 5 (cont)

>Linked List: A LL is a linear data structure constructed like a chain of nodes where each node contains a data field and a reference (link) to the next node in the list.

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Doubly-linked list: Inserting after a given node

> Inserting after a given node can be done by:

>Firstly create a new node (say new_node).

>Now insert the data in the new node.

>Point the next of new_node to the next of prev_node.

>Point the next of prev_node to new_node.

>Point the previous of new_node to prev_node.

>Change the pointer of the new node's previous pointer to new_node.