1.1 Shortcuts in Computation

1. Quicker Counting Methods	Grouping numbers that add up to 5 or 10	73+74+27+26 =(73+27)+(74+26)
	Round off numbers that are close to 5 or 10	73+74+27+26 =75-**2**+75-**1**+25+**2**+25+**1**
2. Sum of numbers that form a pattern	**For patterns where**: numbers increase/decrease by same value	1. rewrite sum in reverse order underneath 2. pair up and sum 3. sums of pairs are the same 4. Since sums are the same, multiple sum by number of pairs 5. Divide by 2
	Example:	Find 2+4+6+...+78+80 2+4+6+...+78+80 80+78+...+6+4+2 82*40=3280 3280/2=1640 2+4+6+...+78+80=**1640**
3. Quicker Muliplication Methods	Remember numbers in their expanded form	3526=3000+500+20+6
	3.1 Multiples of 10	30x25 =3x10x25 =3x250 =750
	3.2 Multiples of 5	25x6 =5x5x6 =5x30 =5x3x10 =15x10 =150

1.2 Number Logic

Properties of Numbers	Primes	factor of 1 and itself only
	Composites	factors other than itself
Divisbility Rules	Divisibility rule of 2	EVEN ends with 0,2,4,6,8
	Divisibility rule of 3	**sum** of its digits divisble by 3
	Divisibility rule of 4	**last 2** digits divisible by 4
	Divisibility rule of 5	ends with 0 or 5
	Divisibility rule of 6	EVEN AND divisible by 3
	Divisible by 8	**last 3** digits divisible by 8
	Divisible by 9	**sum** of its digits divisble by 9
	Divisible by 10	ends with 0
Squared Numbers	NxN=N^2	eg 2x2=2^2=4
Cubed Numbers	NxNxN=N^3	eg 2x2x2=2^3=8

1.3 Developing Patterns and Shortcuts

Factorising Numbers	A number is factorised when expressed as a product of **prime** numbers	250 =2x125 =2x5x25 =2x5x5x5
	Find prime factors	
HCF	**largest** counting number that divides into both exactly	
Highest Common Factor	**method**	1. factorise 2. multiply the factors that are **common** only those factors that have a pair
	example	HCF of 240 and 924 240=**2**x5x**2**x2x**3**x2 924=**3x2**x7x11x**2** HCF=2x3x2=12
LCM	Of all the multiples of the 2 numbers, its the **smallest** multiple they have in common	

1.3 Developing Patterns and Shortcuts (cont)

Lowest Common Multiple	**method**	1. factorise 2. multiply the factors that are common **and** factors they dont have in common
	example	LCM of 120 and 140 120=**2**x**2**x2x**3**x5 140=**2**x**2**x**3**x7 LCM=**2**x**2**x**3**x2x5x7
Question (find multiples)	Jack, Art, Fran and Megan work as volunteers at the local kennel. Jack gives the dogs baths every 4 days. Art cleans out cages every 6 days. Frand feeds the animals in section b every 2 days. Megan helps the receptionist every 3 days. **How many times in 12 weeks will all 4 helpers be at the clinic on the same day?**	**how to solve** Find all the common multiples from 6 days to 84 days (12 weeks) of 4, 6, 2, 3
Question (LCM)	Two buses leave the terminal at **8am**. Bus A takes **60mins** to complete its route and Bus B takes **75mins**. When is the next time the two buses will arrive together at the terminal (if they are on time)?	**how to solve** 1. Find LCM of 60 and 75. 2. Add LCM to 8am

1.3 Developing Patterns and Shortcuts (cont)

Question (HCM and LCM)	Dennis has a choice between two house numbers on Small Street. The two house numbers have their highest common factor of 6. Their least common multiple is 36. One of the house numbers is 12. What is the other number?	**how to solve** work backwards

1.4 Logic Deduction

Logic Deduction Problems	If need to add groups of things, use biggest numbers first	
Question	What is the minimum number of coins needed to make $4.85 from only 5c, 20c, 50c coins	Start with biggest coins first, working through to smaller coins 9x50c=$4.50 1x20c=20c 3x5c=15c
	assume worse case scenario	
	investigate standard case	
	write relations between numbers down	

1.5 Space, Area and Volume

Area of Rectangle	length x width	
Area of Triangle	A = base x height / 2	
Volume of Cube	V = a^3 where a is length of a side	
Volume of Rectangular Prism	V = length x height x depth	
1m	= 100cm	
Finding Area of Rectangular Shapes		
Method 1	Divide shape into rectangles	Find area of each and find total

1.5 Space, Area and Volume (cont)

Method 2	Extend shape into one larger rectangle	1. Find area of larger rectangle (X) 2. Find area of missing rectangle (Y) 3. Larger rectangle (X) - Missing rectangle (Y)

1.6 Equations

Pronumerals	Boxes to store missing numbers	
	Letters to represent unknown numbers	
	Use **x**, **y** and **z**	
Rearranging Equations	= is like a balancing scale	
	solving an equation	**aim** of finding the unknown number
	rearranging equations	**how** to solve an equation
		how if we do something to one side, we need to do the same thing to the other side
		eg. if we **add 3** to one side, we need to **add 3** to the other side
		eg. if we **times by** 3 to one side, we need to **times by** 3 to the other side
	+	-
	x	/
Simultaneous Equations	if there are 2 unknowns, need 2 equations	

1.6 Equations (cont)

1. Solving by Adding and Subtracting Equations	**example**	5x - y = 4 (1) 2x + y = 10 (1)+(2) 7x = 14 x = 2 y = 6
	example	7x + y = 18 (1) 2x + 2y = 12 (2) (1) x 2 14x + 2y = 36 (1a) (1a) - (2) 12x = 24 x = 2 y = 4
2. Solving by Substitution	**method**	1. rearrange one equation for y 2. substitute y into other equation
	example	5x - y = 4 (1) 2x + y = 10 (2) **rearrange (1)** y = 5x - 4 (1a) **substitute (1a) into (2)** 2x + (5x - 4) = 10 x = 2 y = 6

Turning word problems into an equation

Step 1	What are the unknowns?	Give each a letter, **x**, **y**
Step 2	Find the equations to solve	
Step 3	Solve the simultaneous equations	

Example Questions

The quotient of two numbers is 4 and their difference is 39. What is the smaller number of the two

The sum of the ages of Alan and Bill is 25; the sum of the ages of Alan and Carl is 20; the sum of the ages of Bill and Carl is 31. Who is the oldest of the three boys and how old is he?

1.7 Probability, Venn Diagrams and Whodunits

1. Certainty Problems		
Typical Question	Suppose that there are ten black and ten navy socks in your drawer. Your room is dark and you cannot turn on the light. What is the smallest number of socks that you must take out of your drawer to be certain that you have a pair of the same colour?	Basically, to be certain of "an outcome", what is the smallest number of "-actions" required to take
Strategy	**Start from smallest and go up**	
	1 sock	can't be certain
	2 socks	can't be certain
	3 socks	can be certain
2. Certainty Problems with Restrictions		
Typical Question	As above question, but what is the smallest number of socks needed to ensure we get a pair of black socks	Restriction is it must be black socks
Strategy	**Think Worst Case Scenario**	
	Worst case is you could in 10 picks, pick only Navy socks. 2 more picks you'll be certain to get a pair of black socks	
	12 socks	can be certain
Venn Diagrams	circle represents **sets** or groups of *things* that are same	

1.7 Probability, Venn Diagrams and Whodunits (cont)

Example Question	There are 160 students in Year 5. Of these students, 69 walked to school and 57 caught a train to school. If 148 students either walked to school or caught the train, how many students walked **and** caught a train to school?	
	Draw a Venn diagram with a circle for students that walked and students that caught the train	
	Where they overlap, are the number of students that walked and caught the train	
Whodunits		
Strategy	Use a table, with different **characteristics** in columns and **members** of a group in rows	Usually the answer needed are the **characteristics**
Example Question	Martin, Bill and Dave (members of a group) play first base, second base, and third base (characteristics) on their school softball team, but not necessarily in that order. Martin and the third baseman took Dave to the movies yesterday. Martin does not play first base. Who's on first base?	

1.8 Motions, Books, Clocks and Work Problems

1. Motion Problems

distance	= rate x time	
Example Question 1	Two trains leave the same station at the same time, but in **opposite** directions. One train averages 56 km/h and the other averages 64 km/h. How far apart will the trains be when three hours have passed?	
Strategy	Step 1	Whats the distance after 1hr? (Draw a diagram)
		56km + 64km = 120km
		56km/hr + 64km/hr = 120km/hr
	Step 2	Whats the distance after 3hrs?
		120km x 3 = 360km
	if **opposite** direction,	**add**
Example Question 2	Suppose that these two trains start from the same station at the same time, this time in the same direction. How far apart will the fronts of the trains be at the end of the three hours?	
	Step 1	Whats the distance after 1hr? (Draw a diagram)
		64km/hr - 56km/hr = 8km/hr
		64km - 56km = 8km
	Step 2	Whats the distance after 3hrs?
		8km x 3 = 24km
	if **opposite** direction,	**subtract**

2. Book Problems

look at the structure of counting numbers used for book pages

1.8 Motions, Books, Clocks and Work Problems (cont)

Example Question	A printer uses an old-style printing press and needs one piece of type for each digit in the page numbers of a book. How many 2s will the printer need to print page numbers from 1 to 250	
consider the numbers place by place		
	number of times 2s appear in the 1s place	25
	number of times 2s appear in the 10s place	30
	number of times 2s appear in the 100s place	51
	answer	=25+30+51 =106

3. Clock Problems

elapse time	amount of time that has passed	
solve using	facts about time	
Example Question	A certain clock gains one minute of time every hour. If the clock shows the correct time now, in how many hours will it next show the correct time again without regard to am or pm?	
Fact 1	A clock that has stopped	Will show the correct time every 12 hrs. As it stopped at 6.03am on Monday. It was correct at the time it stopped. It will be correct again, when the time is at 6.03pm

1.8 Motions, Books, Clocks and Work Problems (cont)

Fact 2	The clock in the problem must gain 12 hours to show correct time again	
thus	12 hrs	=60mins x 12 = 720mins
thus	as clock gains 1 min in 1hr	the clock will gain 720min in 720hrs
		720/24=30days
4. Work Problems		
solving using	fractional parts of whole numbers and draw diagrams	
Example Question	Paul can do a certain job in 3hrs and John can do the same job in 2hrs. At these rates, how long would it take Paul and John to do this job if they work together	
Strategy	Step 1	Draw a diagram for Paul and John. Fractional parts done in each hour
	Step 2	Using the diagram, in one hour they can complete 1/3 + 1/2 = 5/6 of the job
	Step 3	Work out how long to complete job
		1/5 of job left
		60min / 5 = 12mins to complete 1/5 of job
	answer	=1hr 12mins

1.9 Problem Solving Strategies

1. Drawing a picture or diagram	
Example Question	The lengths of three rods are 5cm, 7cm, and 15cm. How can you use these rods to measure a length of 13cm?
2. Making an organised list	

1.9 Problem Solving Strategies (cont)

Example Question	Five students hold a chess tournament. Each of the students plays each of the other students just once. How many different games are played?
3. Making a table	
Example Question	Two dice both have faces numbered from 1 through to 6. Suppose that you role the two dice. What is the probability of rolling a sum of 8 in the uppermost faces?
4. Solving a simpler related problem	
Example Question	The houses on Thomas Street are numbered consecutively from 1 to 150. How many house numbers contain at least one digit 7?
5. Finding a pattern	
Example Question	What is the sum of the following series of numbers?
6. Guessing and Checking	
Example Question	Arrange the counting numbers from 1 to 6 in the circles so that the sum of the numbers along each side of the triangle is 10.

1.10 Problem Solving Strategies

1. Acting out the problem	
Example Question	Suppose that you buy a rare stamp for $16, sell it for $22, buy it back for $30, and finally sell it for $35. How much money did you make or lose?
2. Working backwards	
Example Question	At the end of a school day, a teacher had 15 crayons left. The teacher remembered giving out 13 of all her crayons in the morning, getting 8 back at recess, and giving out 9 crayons after lunch. How many crayons did the teacher have at the start of the day?

1.10 Problem Solving Strategies (cont)

3. Writing an Equation	
Example Question	The triple of what number is sixteen greater than the number?
4. Changing your point of view	Change your approach
	Are you assuming something thats not in the question
Example Question	Draw four continuous line segments through the nine dots
5. Using Reasoning	
Example Question	A school has 731 students. Prove that there must be at least 3 students who have the same birthday.
6. Miscellaneous	
Example Question	Three apples and two pears cost 78 cents. But two apples and three pears cost 82 cents. What is the total cost of one apple and one pear?

2.1 Logical Approach to Problem Solving

4 Steps to Problem Solving		
Step 1	Understand the problem	
Step 2	Develop a plan	**choose a problem solving strategy**
Step 3	Carry out the plan	
Step 4	Reflect	
Mathematical Terms used in the Olympiad		
Standard Form	1358	
Expanded Form	1x1000+3x100+5x10+8x1	
Exponential Form	1x10^3+3x10^2+5x10+8x1	
Whole numbers	0,1,2,3,...	
Counting numbers	1,2,3,...	
Divisibility	A is divisible by B, if B divides into A with zero remainder	If so, B is a factor of A

2.1 Logical Approach to Problem Solving (cont)

Prime number	counting number greater than 1, which is divisible only by itself and
Composite number	counting number greater than 1 which is divisible by a counting number other than 1 and itself
A number is factored completely	when it is a product of prime numbers
Order of Operation	BODMAS
common or simple fraction	a/b where a and b are whole numbers and b is no zero
unit fraction	common fraction with a numerator of 1
proper fraction	a/b where a < b
improper fraction	a/b where a > b
complex fraction	numerator or denominator contains a fraction
20th century	100 year period 1901-2000 inclusive
average of a set of N numbers	sum of the N numbers divided by N
acute angle	less than 90 degrees
right angle	90 degrees
obtuse angle	greater than 90 degrees
straight angle	180 degrees
reflex angle	more than 180 degrees and less than 360 degrees
scalene triangle	no equal angles
isosceles triangle	2 equal angles
equilateral	3 equal angles
right-angled	90 angle

2.1 Logical Approach to Problem Solving (cont)

congruent shapes	shapes on the same plane whose sides and angles are the same

2.2 Types of Problems

1. Translation Problems	translate word sentences to mathematical sentences
Example Question	Farmer Joe bought 2 bags of feed for $4 each and 1 bag of feed for $3. How much did the feed bags cost altogether?
2. Application Problems	'real-world' problems, usually involve calculations with money, to find **discounts**, **profits** or **cost** of items
Example Question	Shop A is offering a 10% discount on 34cm colour TV sets priced normally at $379. Meanwhile Shop B is offering 15% discount on the same sets priced normally at $409. Which shop should you purchase the TV from?
3. Process Problems	Usually require using general problem solving steps and specific strategies. May use short-cuts when aware of patterns
Example Question	The first 4 triangular numbers are 1, 3, 6, 10. What will the 10th triangular number be?
4. Puzzle Problems	like riddles
Example Question	Three Australian students who were born in different countries have last names Brown, Black and Bright. Their first names are Jim, John and Jane but not necessarily in that order. Using the information below can you determine the full name of each student? Brown was born in Australia Bright has never been to Malaysia Jane was born in England Jim was born in Malaysia