### Variable key

**Where:**
- \( FV \) = Future value of an investment
- \( PV \) = Present value of an investment (the lump sum)
- \( r \) = Return or interest rate per period (typically 1 year)
- \( n \) = Number of periods (typically years) that the lump sum is invested
- \( PMT \) = Payment amount
- \( CF_n \) = Cash flow steam number
- \( m \) = \# of times per year \( r \) compounds

### Equation guide

#### Future value of a lump sum:
\[
FV = PV \times (1 + r)^n
\]
- Future-value factor (FVF) table
- Excel future value formula \( FV=PV \times r^n + \sum_{i=1}^{n} \frac{PV \times r^i}{(1 + r)^i} \)

#### Future Value of an Ordinary Annuity
\[
FV = PMT \times \left[ \frac{(1 + r)^n - 1}{r} \right]
\]

#### Future Value of an Annuity Due
\[
FV \text{ (annuity due)} = PMT \times \left[ \frac{(1 + r)^n - 1}{r} \right] \times (1 + r)
\]

#### Future Value of Cash Flow Streams
\[
FV = CF_1 \times (1 + r)^n + CF_2 \times (1 + r)^{n-1} + ... + CF_n \times (1 + r)^{-n}
\]

### Present Value of a lump sum in future
\[
PV = FV / (1 + r)^n = FV \times \left[ \frac{1}{(1 + r)^n} \right]
\]
- Present-value factor (FVF) table
- Excel present value formula \( PV=\frac{FV}{r} \times \left[ \frac{1}{(1 + r)^n} \right] \)

#### Present Value of an Ordinary Annuity
\[
PV = PMT / r \times \left[ \frac{1 - 1 / (1 + r)^n}{r} \right]
\]

#### Present Value of an Annuity Due
\[
PV \text{ (annuity due)} = PMT / r \times \left[ \frac{1 - 1 / (1 + r)^n}{r} \right] \times (1 + r)
\]

#### Present Value of Cash Flow Streams
\[
PV = [CF_1 \times \left[ \frac{1}{(1 + r)^1} \right] + CF_2 \times \left[ \frac{1}{(1 + r)^2} \right] + ... + CF_n \times \left[ \frac{1}{(1 + r)^n} \right]]
\]

### Present Value of a Growing Perpetuity
Most cash flows grow over time

This formula adjusts the present value of a perpetuity formula to account for expected growth in future cash flows

Calculate present value \( PV \) of a stream of cash flows growing forever \((n = \infty)\) at the constant annual rate \( g \)
\[
PV = CF_1 / r - g \quad r > g
\]

### Stated Versus Effective Annual Interest Rates (cont)

Maximum effective annual rate for a stated annual rate occurs when interest compounds continuously
\[
EAR = (1 + r/m)^m - 1
\]

Compounding continuously: \( EAR \) (continuous compounding) = \( e^r - 1 \)

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### Deposits Needed to Accumulate a Future Sum

Determine the annual deposit necessary to accumulate a certain amount of money at some point in the future

E.g. house deposit

Can be derived from the equation for finding the future value of an ordinary annuity

Can also be used to calc required deposit
\[
PMT = FV \times \left[ \frac{1 - 1 / (1 + r)^n}{r} \right]
\]

Once this is done substitute the known values of \( FV, r, \) and \( n \) into the righthand side of the equation to find the annual deposit required.

### Loan Amortization

A borrower makes equal periodic payments over time to fully repay a loan

E.g. home loan

**Uses**
- Total $ of loan
- Term of loan
- Frequency of payments
- Interest rate

Finding a level stream of payments (over the term of the loan) with a present value calculated at the loan interest rate equal to the amount borrowed

**Loan amortization schedule** Used to determine loan amortisation payments and the allocation of each payment to interest and principal
### Loan Amortization (cont)

**Portion of payment representing interest declines over the repayment period, and the portion going to principal repayment increases**

\[
PMT = \frac{PV / (1 / r x [1 - 1 / (1 + r)]^n)}{}
\]

### Concept of future value

Apply simple interest, or compound interest to a sum over a specified period of time.

Interest might compound: annually, semiannually, quarterly, and even continuous compounding periods

**Future value** value of an investment made today measured at a specific future date using compound interest.

**Compound interest** is earned both on principal amount and on interest earned

**Principal** refers to amount of money on which interest is paid.

### Important to understand

After 30 years @ 5% a $100 principle account has:
- Simple Interest: balance of $250.
- Compound interest: balance of $432.19

**FV = PV x (1 + r)^n**

### The Power of Compound Interest

![Future Value of One Dollar](chart)

The figure shows that the future value of $1 increases over time if interest is earned at a constant interest rate. Notice that each line is steeper than the one that immediately precedes it. This is the presence of compound interest. For this same reason, the future value becomes larger as the compounding frequency gets faster or the interest rates increase.

### Present value

Used to determine what an investor is willing to pay today to receive a given cash flow at some point in future.

Calculating present value of a single future cash payment

Depends largely on investment opportunities of recipient and timing of future cash flow

**Discounting** describes process of calculating present values

- Determines present value of a future amount, assuming an opportunity to earn a return (r)
- Determine PV that must be invested at r today to have FV, n from now
- Determines present value of a future amount, assuming an opportunity to earn a given return (r) on money.

We lose opportunity to earn interest on money until we receive it

To solve, inverse of compounding interest

PV of future cash payment declines longer investors wait to receive

Present value declines as the return (discount) r rises.

E.g. value now of $100 cash flow that will come at some future date is less than $100

\[
PV = \frac{FV}{(1 + r)^n} = FV \times \left(\frac{1}{1 + r^n}\right)
\]

### Special applications of time value

Use the formulas to solve for other variables

- **Cash flow** CF or PMT
- **Interest / Discount rate** r
- **Number of periods** n

Common applications and refinements

- Compounding more frequently than annually
- Stated versus effective annual interest rates
- Calculation of deposits needed to accumulate a future sum
- Loan amortisation

### Compounding More Frequently Than Annually

Financial institutions compound interest semiannually, quarterly, monthly, weekly, daily, or even continuously.

The more frequently interest compounds, the greater the amount of money that accumulates

**Semiannual compounding**

Compounds twice per year

**Quarterly compounding**

Compounds 4 times per year

**m values:**

- Semiannual: 2
- Quarterly: 4
- Monthly: 12
- Weekly: 52
- Daily: 365

**Continuous Compounding**

\[m = \infty\]

### The Power of Discounting

![Present Value of One Dollar](chart)

The present value of $100 rises at the same rate as dollar interest. Similarly, the longer a sum must wait to receive a $100 payment, the lower the present value of that payment.

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### Compounding More Frequently Than Annually (cont)

- **e** = irrational number \( \approx 2.7183.13 \)

General equation: \( FV = PV \times (1 + \frac{r}{m})^{m \times n} \)

Continuous equation: \( FV \) (continuous compounding) = \( PV \times (e^{rn}) \)

### Finding the Future Value of an Annuity Due

- Slight change to those for an ordinary annuity
- Payment made at beginning of period, instead of end
- Ears interest for 1 period longer
- Ears more money over the life of the investment

\[
FV \text{ (annuity due)} = \text{PMT} \times \left[ \frac{(1 + r)^n - 1}{r} \right] \times (1 + r)
\]

### Present Value of an Annuity Due

- Similar to mixed stream / ordinary annuity
- Discount each payment and then add up each term
- Cash flow realised 1 period earlier
- Annuity due has a larger present value than ordinary annuity

\[
PV \text{ (annuity due)} = \frac{\text{PMT}}{r} \times \left[ 1 - \frac{1}{(1 + r)^n} \right]
\]

### Present Value of a Perpetuity

- Level or growing cash flow stream that continues forever
- Level = infinite life
- Simplest modern example = preferred stock
- Preferred shares promise investors a constant annual (or quarterly) dividend payment forever
- express the lifetime (n) of this security as infinity (\( \omega \))

\[
PV = \frac{\text{PMT}}{r}
\]

### Future Value of Cash Flow Streams

Evaluate streams of cash flows in future periods.

- **Mixed stream** = a series of unequal cash flows reflecting no particular pattern
- **Annuity** = A stream of equal periodic cash flows
- More complicated than calculating future or present value of a single cash flow, same basic technique.

Shortcuts available to evaluate an annuity

- AKA terminal value
- \( FV \) of any stream of cash flows at EOY = sum of \( FV \) of individual cash flows in that stream, at EOY

Each cash flow earns interest, so future value of stream is greater than a simple sum of its cash flows

\[
FV = CF_1 \times (1 + r)^{n-1} + CF_2 \times (1 + r)^{n-2} + \ldots + CF_n \times (1 + r)^{-n}
\]

### Future Value of an Ordinary Annuity

- Two basic types of annuity:
  - **Ordinary annuity** = payments made into it at end of each period
  - **Annuity due** = payments made into it at the beginning of each period (arrives 1 year sooner)

So, future value of an annuity due always greater than ordinary annuity

Future value of an ordinary annuity can be calculated using same method as a mixed stream

\[
FV = \text{PMT} \times \left[ \frac{(1 + r)^n - 1}{r} \right]
\]

### Present Value of a Mixed Stream

- Present values of cash flow streams that occur over several years
- Might be used to:
  - Value a company as a going concern
  - Value a share of stock with no definite maturity date

\[
PV = \sum CF_n \times \frac{1}{(1 + r)^n}
\]

### Present Value of an Ordinary Annuity

- Similar to mixed stream
- Discount each payment and then add up each term

\[
PV = \frac{\text{PMT}}{r} \times \left[ 1 - \frac{1}{(1 + r)^n} \right]
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### Present Value of an Annuity Due

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