Time value of money Cheat Sheet

by Natalie Moore (NatalieMoore) via cheatography.com/19119/cs/11141/

Variable key

Where:	
FV	= Future value of an investment
PV	= Present value of an investment (the lump sum)
r	= Return or interest rate per period (typically 1 year)
n	= Number of periods (typically years) that the lump sum is invested
PMT	= Payment amount
CFn	= Cash flow steam number
m	= # of times per year r compounds

Equation guide

Future value of a lump sum:

$FV = PV \times (1 + r)^n$

- Future-value factor (FVF) table
- Excel future value formula FV=
- Compound interest. Formula for simple interest is PV + (n x (PV x r))

Future Value of an Ordinary Annuity

$FV = PMT \times \{ [(1 + r)^n - 1] / r\}$

Future Value of an Annuity Due

$FV \text{ (annuity due)} = PMT \times \{ [(1 + r)^n - 1] / r \} \times (1 + r)$

Future Value of Cash Flow Streams

$FV = CF1 \times (1 + r)^{n-1} + CF2 \times (1 + r)^{n-2} + \ldots + CFn \times (1 + r)^{n-n}$

Present value of a lump sum in future

$PV = FV / (1 + r)^n = FV \times [1 / (1+ r)^n]$

- Present-value factor (FVF) table
- Excel present value formula PV=

Equation guide (cont)

Present Value of a Mixed Stream

$PV = [CF1 \times 1 / (1 + r)^1] + [CF2 \times 1 / (1 + r)^1] + \ldots + [CFn \times 1 / (1 + r)^1]$

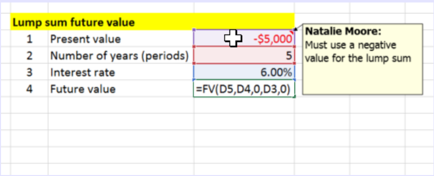
Present Value of an Ordinary Annuity

$PV = PMT/r \times [1 - 1 / (1 + r)^n]$

Present Value of Annuity Due

$PV \text{ (annuity due)} = PMT/r \times [1 - 1 / (1 + r)^n] \times (1 + r)$

Lump sum future value in excel



Lump sum future value		
1	Present value	-$5,000
2	Number of years (periods)	5
3	Interest rate	6.00%
4	Future value	=FV(D5,D4,0,D3,0)

Present Value of a Growing Perpetuity

Most cash flows grow over time

This formula adjusts the present value of a perpetuity formula to account for expected growth in future cash flows

Calculate present value (PV) of a stream of cash flows growing forever (n = ∞) at the constant annual rate g

$PV = CF1 / r - g \quad r > g$

Loan Amortization

A borrower makes equal periodic payments over time to fully repay a loan

E.g. home loan

Uses

- Total $ of loan
- Term of loan

Loan Amortization (cont)

- Frequency of payments
- Interest rate

Finding a level stream of payments (over the term of the loan) with a present value calculated at the loan interest rate equal to the amount borrowed

Loan amortization schedule Used to determine loan amortisation payments and the allocation of each payment to interest and principal

Portion of payment representing interest declines over the repayment period, and the portion going to principal repayment increases

$PMT = PV / \{1 / r \times [1 - 1 / (1 + r)^n] \}$

Deposits Needed to Accumulate a Future Sum

Determine the annual deposit necessary to accumulate a certain amount of money at some point in the future

E.g. house deposit

Can be derived from the equation for fi nding the future value of an ordinary annuity

Can also be used to calc required deposit

$PMT = FV \{[(1 + r)^n - 1] / r\}$

Once this is done substitute the known values of FV, r, and n into the righthand side of the equation to find the annual deposit required.

Stated Versus Effective Annual Interest Rates

Make objective comparisons of loan costs or investment returns over different compounding periods

Stated annual rate is the contractual annual rate charged by a lender or promised by a borrower

Stated Versus Effective Annual Interest Rates (cont)

Effective annual rate (EAR) AKA the true annual return, is the annual rate of interest actually paid or earned

- Reflects the effect of compounding frequency
- Stated annual rate does not

Maximum effective annual rate for a stated annual rate occurs when interest compounds continuously

EAR = $(1 + r/m)^m - 1$

Compounding continuously: **EAR (continuous compounding) = $e^r - 1$**

Concept of future value

Apply simple interest, or compound interest to a sum over a specified period of time.

Interest might compound: annually, semiannual, quarterly, and even continuous compounding periods

Future value value of an investment made today measured at a specific future date using compound interest.

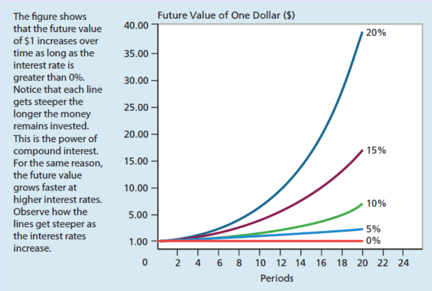
Compound interest is earned both on principal amount and on interest earned

Principal refers to amount of money on which interest is paid.

Important to understand
After 30 years @ 5% a $100 principle account has:
- Simple Interest: balance of $250.
- Compound interest: balance of $432.19

FV = PV x $(1 + r)^n$

The Power of Compound Interest



Future Value of One Dollar

Present value

Used to determine what an investor is willing to pay today to receive a given cash flow at some point in future.

Calculating present value of a single future cash payment

Depends largely on investment opportunities of recipient and timing of future cash flow

Discounting describes process of calculating present values

- Determines present value of a future amount, assuming an opportunity to earn a return (r)
- Determine PV that must be invested at r today to have FV, n from now
- Determines present value of a future amount, assuming an opportunity to earn a given return (r) on money.

We lose opportunity to earn interest on money until we receive it

To solve, inverse of compounding interest

PV of future cash payment declines longer investors wait to receive

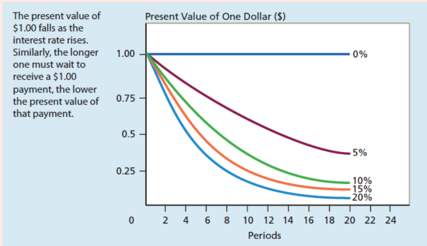
Present value (cont)

Present value declines as the return (discount) rises.

E.g. value now of $100 cash flow that will come at some future date is less than $100

PV = FV / $(1 + r)^n$ = FV x [1 / $(1+ r)^n$]

The Power of Discounting



Special applications of time value

Use the formulas to solve for other variables

-	Cash flow	CF or PMT
-	Interest / Discount rate	r
-	Number of periods	n

Common applications and refinements

- Compounding more frequently than annually
- Stated versus effective annual interest rates
- Calculation of deposits needed to accumulate a future sum
- Loan amortisation

Compounding More Frequently Than Annually

Financial institutions compound interest semiannually, quarterly, monthly, weekly, daily, or even continuously.

The more frequently interest compounds, the greater the amount of money that accumulates

Semiannual compounding

Compounds twice per year

Quarterly compounding

Compounds 4 times per year

m values:

Semiannual	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

Continuous Compounding

m = infinity

e = irrational number ~2.7183.[13]

General equation: **$FV = PV \times (1 + r / m)^{mxn}$**

Continuous equation: **FV (continuous compounding) = $PV \times (e^{rxn})$**

Future Value of Cash Flow Streams

Evaluate streams of cash flows in future periods.

Two types:

Mixed stream = a series of unequal cash flows reflecting no particular pattern

Annuity = A stream of equal periodic cash flows

More complicated than calc future or present value of a single cash flow, **same basic technique**.

Shortcuts available to eval an annuity

Future Value of Cash Flow Streams (cont)

AKA terminal value

FV of any stream of cash flows at EOY = sum of FV of individual cash flows in that stream, at EOY

Each cash flow earns interest, so future value of stream is greater than a simple sum of its cash flows

$FV = CF1 \times (1 + r)^{n-1} + CF2 \times (1 + r)^{n-2} + ... + CFn \times (1 + r)^{n-n}$

Future Value of an Ordinary Annuity

Two basic types of annuity:

Ordinary annuity = payments made into it at end of each period

Annuity due = payments made into it at the beginning of each period (arrives 1 year sooner)

So, future value of an annuity due always greater than ordinary annuity

Future value of an ordinary annuity can be calculated using same method as a mixed stream

$FV = PMT \times \{ [(1 + r)^{n} - 1] / r\}$

Finding the Future Value of an Annuity Due

Slight change to those for an ordinary annuity

Payment made at beginning of period, instead of end

Earns interest for 1 period longer

Earns more money over the life of the investment

FV (annuity due) = $PMT \times \{ [(1 + r)^{n} - 1] / r \} \times (1 + r)$

Present Value of Cash Flow Streams

Present values of cash flow streams that occur over several years

Might be used to:

- Value a company as a going concern
- Value a share of stock with no definite maturity date

= sum of the present values of CFn

Perpetuity: A level or growing cash flow stream that continues forever

Same technique as a lump sum

Present Value of a Mixed Stream = Sum of present values of individual cash flows

Mixed stream:
$PV = [CF1 \times 1 / (1 + r)^{1}] + [CF2 \times 1 / (1 + r)^{1}] + ... + [CFn \times 1 / (1 + r)^{1}]$

Present value of an ordinary annuity

Present Value of an Ordinary Annuity

Similar to mixed stream

Discount each payment and then add up each term

$PV = PMT/r \times [1 - 1 / (1 + r)^{n}]$

Present Value of Annuity Due

Similar to mixed stream / ordinary annuity

Discount each payment and then add up each term

Cash flow realised 1 period earlier

Annuity due has a larger present value than ordinary annuity

PV (annuity due) = $PMT/r \times [1 - 1 / (1 + r)^{n}] \times (1 + r)$

Present Value of a Perpetuity

Level or growing cash fl ow stream that continues forever

Level = infinite life

Simplest modern example = prefered stock

Preferred shares promise investors a constant annual (or quarterly) dividend payment forever

- express the lifetime (n) of this security as infi nity (∞)

PV = PMT x 1/r = PMT/r