Basic of Mathematics used in Physics Cheat Sheet

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Quadratic Equation

Roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots $x_1 + x_2 = -\frac{b}{a}$;

Product of roots $x_1 x_2 = \frac{c}{a}$

For real roots, $b^2 - 4ac \geq 0$

For imaginary roots, $b^2 - 4ac < 0$

Logarithm

$\log_{10} N = x \Rightarrow 10^x = N$

$\log_b N = \log_b a * \log_a N$

$\log_b 1 = 0$, $\log_a a = 1$

$\log mn = \log m + \log n$ $\qquad$ $\log \frac{m}{n} = \log m - \log n$

$\log m^n = n \log m$ $\qquad$ $\log_e m = 2.303 \log_{10} m$

$\log 2 = 0.3010$ $\qquad$ $\log 3 = 0.4771$

Arithmetic Progression

$a, a+d, a+2d, a+3d, \ldots\ldots a+(n-1)d$

here d = common difference

Sum of n terms $S_n = \frac{n}{2}[2a+(n-1)d]$

n^{th} term, $a_n = a + (n-1)d$

Geometric Progression

$a, ar, ar^2, ar^3, \ldots\ldots$ here, r = common ratio

n^{th} term, $a_n = a.r^{n-1}$

Sum of n terms $S_n = \frac{a(1-r^n)}{1-r}$

Sum of ∞ terms $S_\infty = \frac{a}{1-r}$ [where $|r| < 1$]

Cosine Rule

$\cos A = \frac{b^2+c^2-a^2}{2bc}$, $\cos B = \frac{c^2+a^2-b^2}{2ca}$, $\cos C = \frac{a^2+b^2-c^2}{2ab}$

Sine Rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Binomial Theorem

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{6}x^3 + \ldots$

$(1-x)^n = 1 - nx + \frac{n(n-1)}{2}x^2 - \frac{n(n-1)(n-2)}{6}x^3 + \ldots$

If $x \ll 1$ then $(1+x)^n \approx 1 + nx$ & $(1-x)^n \approx 1 - nx$

Trigonometry I

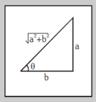
2π radian $= 360° \Rightarrow 1$ rad $= 57.3°$

$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$ | $\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$ | $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$\cot\theta = \frac{\text{base}}{\text{perpendicular}}$ | $\sec\theta = \frac{\text{hypotenuse}}{\text{base}}$ | $\text{cosec}\,\theta = \frac{\text{hypotenuse}}{\text{perpendicular}}$

$\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$ | $\cos\theta = \frac{b}{\sqrt{a^2+b^2}}$ | $\tan\theta = \frac{a}{b}$

$\text{cosec}\theta = \frac{1}{\sin\theta}$ | $\sec\theta = \frac{1}{\cos\theta}$ | $\cot\theta = \frac{1}{\tan\theta}$



Trigonometry II

$\sin^2\theta + \cos^2\theta = 1$ | $1 + \tan^2\theta = \sec^2\theta$ | $1 + \cot^2\theta = \text{cosec}^2\theta$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$ $\qquad$ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ $\qquad$ $\sin 2A = 2\sin A\cos A$

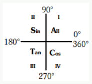
$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$ $\qquad$ $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$ $\qquad$ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$2\cos A\cos B = \cos(A-B) + \cos(A+B)$ $\qquad$ $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

Trigonometry III



Trigonometry IV

θ	0° (0)	30° (π/6)	45° (π/4)	60° (π/3)	90° (π/2)	120° (2π/3)	135° (3π/4)	150° (5π/6)	180° (π)	270° (3π/2)	360° (2π)
sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	**0**
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	0	**1**
tanθ	**0**	$\frac{1}{\sqrt{3}}$	**1**	$\sqrt{3}$	**∞**	$-\sqrt{3}$	**−1**	$-\frac{1}{\sqrt{3}}$	**0**	**∞**	**0**

Trigonometry V

$\sin(90° + \theta) = \cos\theta$
$\cos(90° + \theta) = -\sin\theta$
$\tan(90° + \theta) = -\cot\theta$

$\sin(180° + \theta) = -\sin\theta$
$\cos(180° + \theta) = -\cos\theta$
$\tan(180° + \theta) = \tan\theta$

$\sin(180° - \theta) = \sin\theta$
$\cos(180° - \theta) = -\cos\theta$
$\tan(180° - \theta) = -\tan\theta$

$\sin(270° - \theta) = -\cos\theta$
$\cos(270° - \theta) = -\sin\theta$
$\tan(270° - \theta) = \cot\theta$

$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$

$\sin(270° + \theta) = -\cos\theta$
$\cos(270° + \theta) = \sin\theta$
$\tan(270° + \theta) = -\cot\theta$

$\sin(90° - \theta) = \cos\theta$
$\cos(90° - \theta) = \sin\theta$
$\tan(90° - \theta) = \cot\theta$

$\sin(360° - \theta) = -\sin\theta$
$\cos(360° - \theta) = \cos\theta$
$\tan(360° - \theta) = -\tan\theta$

For Small Theta

$\sin\theta \approx \theta$ $\qquad$ $\cos\theta \approx 1$ $\qquad$ $\tan\theta \approx \theta$ $\qquad$ $\sin\theta \approx \tan\theta$

Average of a Varying Quantity

If $y = f(x)$ then $\langle y \rangle = \bar{y} = \frac{\int_{x_1}^{x_2} y\,dx}{\int_{x_1}^{x_2} dx} = \frac{\int_{x_1}^{x_2} y\,dx}{x_2 - x_1}$

Integration

C = Arbitrary constant, k = constant

- $\int f(x)dx = g(x) + C$
- $\frac{d}{dx}(g(x)) = f(x)$
- $\int kf(x)dx = k\int f(x)dx$
- $\int (u+v+w)dx = \int u\,dx + \int v\,dx + \int w\,dx$
- $\int e^x dx = e^x + C$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
- $\int \frac{1}{x}dx = \ell nx + C$
- $\int \sin x\,dx = -\cos x + C$
- $\int \cos x\,dx = \sin x + C$
- $\int e^{\alpha x+\beta}dx = \frac{1}{\alpha}e^{\alpha x+\beta} + C$
- $\int (\alpha x+\beta)^n dx = \frac{(\alpha x+\beta)^{n+1}}{\alpha(n+1)} + C$

Differentiation

- $y = x^n \rightarrow \frac{dy}{dx} = nx^{n-1}$ • $y = \ell nx \rightarrow \frac{dy}{dx} = \frac{1}{x}$
- $y = \sin x \rightarrow \frac{dy}{dx} = \cos x$ • $y = \cos x \rightarrow \frac{dy}{dx} = -\sin x$
- $y = e^{\alpha x+\beta} \rightarrow \frac{dy}{dx} = \alpha e^{\alpha x+\beta}$ • $y = uv \rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
- $y = f(g(x)) \Rightarrow \frac{dy}{dx} = \frac{df(g(x))}{dg(x)} \times \frac{d(g(x))}{dx}$
- $y = k$(constant) $\Rightarrow \frac{dy}{dx} = 0$
- $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Maxima and Minima

For maximum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} = -ve$

For minimum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} = +ve$

Componendo & Dividendo Theorem

If $\frac{p}{q} = \frac{a}{b}$ then $\frac{p+q}{p-q} = \frac{a+b}{a-b}$