

Little's Law

WORK IN PROGRESS WIP = TH*CT

Cycle Time CT WIP/TH

Throughput TH WIP/CT

Bottleneck Rate rb =1/Max Avg Processing Time

Raw Processing Time To Sum of Avg Processing Time

Critical WIP Wo rb*To

BEST CASE PERFORMANCE

CT BEST if $w \leq Wo$ To
 otherwise W/rb

TH BEST if $w < Wo$ w/To
 otherwise rb

WORST CASE PERFORMANCE

CTworst =w*To

THworst 1/To

PRACTICAL WORST CASE

CTpwc To+((w-1)/rb)

THpwc (w/(Wo+w-1))rb

Sample Midterm Qsle Midterm Qs (Cont)

b) A company supplying seats to an auto assembly plant sends trucks to its customer at an average rate of 6 trucks per day. Given the travel time to the customer is an average of three days, what is the average number of trucks in transit at any given time?

TH = 6 trucks/day

CT = 3 days

WIP = TH x CT = 18 trucks

PREEMPTIVE ONLY

Natural Proc. Time to

STD of Nat. Proc. Time σ_o

SCV of Nat. Proc. Time co^2 σ_o^2/to^2

STD of Nat. Proc. Time mf

Mean Time to Repair mr

STD of Time to Repair σ_r

Mean Availability A $mf/(mf+mr)$

SCV of Time to Repair cr^2 σ_r^2/mr

Mean Eff. Time with Preemptive Outages $te(Po)$ to/A

SCV of Eff. Time with Preemptive Outages $ce(PO)^2$ $co^2+(1+cr^2)A(1-A)^{-1}-mr/to$

PREEMPTIVE PLUS NON PREEMPTIVE

Mean batch size Ns

Mean batch size ts

STD of Setup Time σ_s

Mean Eff. Time with Preemptive Outages te $te(PO)+ts/Ns$

std. dev. Squared of eff. Time σ_e^2 $te(po)^2 \times cd(po)^2+(r-s^2/Ns) + (Ns-1/Ns^2) \times ts^2$

SCV of Eff. Time with Preemptive Outages ce^2 σ_e^2/te^2

Mean Utilization u te/ta

SCV of interarrival times ca^2

SCV of interdeparture times cd^2 $1 + (1-u^2) \times (ca^2-1) + u^2(c-e^2-1)$

PREEMPTIVE PLUS NON PREEMPTIVE (cont)

CTQ $(ca^2 + ce^2)/2 \times u/(1-u) \times te$

CT CTQ + te

WIP CT/ta

WIP CT/ta

Sample Midterm Qsle Midterm Qs (Cont)

b) A company supplying seats to an auto assembly plant sends trucks to its customer at an average rate of 6 trucks per day. Given the travel time to the customer is an average of three days, what is the average number of trucks in transit at any given time?

TH = 6 trucks/day

CT = 3 days

WIP = TH x CT = 18 trucks

Sample Midterm Qs

SCV of Effective processing time $ce^2 = ((\sigma_o^2 + (\sigma_s^2/N) + (Ns-1)/Ns^2 \times ts^2)/te$

Utilization $u = (te/ta) = (te(np)/ta)$

Utilization (coffee shop) $u = ra/re=te/ta$

WIP (M/M/1) $u/(1-u)$

WIP (M/M/1) $u/(1-u)$

CT (M/M/1) $te/(1-u)$

Sample Midterm Qs (Cont)

3. a) Compute the average cycle time at machine 1.

$$CTq1 = (Ca^2 + Ce^2)/2 \times (u/(1-u)) \times te$$

$$u = (te/ta)$$

b) Compute the mean and coefficient of variation of the time between departures from Machine 1.

$$ta(2) = td(1) = ta(1)$$

$$cd^2 = 1 + (1-u^2) \times (ca^2 - 1) + u^2(ce^2 - 1)$$

$$u2 = te2/ta2 = 18/22$$

c) Compute the average cycle time at machine 2

$$CT(2) = CTq(2) + te(2) \text{ (note use } u2 \text{ to calculate } CTq(2))$$

d) Calculate the total CT and WIP of the system (combining machine 1 and 2).

$$\text{Total CT} = CT(1) + CT(2)$$

From Little's Law

$$\text{Total WIP} = CT \times TH = CT/ta$$

e) Now suppose the line must produce both products in equal proportion, i.e., one unit of Product 1 for each unit of Product 2. Estimate the bottleneck rate and raw process time of the line under this product mix.

Hint: Think about the what the average processing time will be at each machine.

Processing time at $(M1 + M2)/\#$ of machines
(2 calculations ..1 for each product)

$$rb = 1/\text{largest processing}$$

$$to = to1 + to2 \text{ (answer from processing time)}$$

Kendall Notation

M Memoryless or exponential

D Deterministic

G General

G/G/1 not exponential, gives approx CT and CTq

M/M/1 exponential, infinite source
population unlimited queue length

M/M/1 Queuing

$$WIP = u/(1-u)$$

$$CT = WIP/ra = te/(1-u)$$

$$CTq = CT - te = (u \times te)/(1-u)$$

$$WIPq = raCTq = u^2/(1-u)$$

G/G/1 QUE

$$CT = ((ca^2 + ce^2)/2)(u/(1-u)) \times te$$

M/M/1/b

$$WIP = u/(1-u) - ((b+1)u^{b+1}) / (1-u^{b+1})$$

$$TH = ((1-u^b) / (1-u^{b+1})) \times ra$$

• Smaller buffer sizes bring greater losses relative to uncapacitated system



By musikdr

cheatography.com/musikdr/

Published 3rd October, 2019.

Last updated 3rd October, 2019.

Page 2 of 2.

Sponsored by **Readable.com**

Measure your website readability!

<https://readable.com>