

Little's Law

WORK IN PROGRESS	WIP = TH*CT
Cycle Time	CT = WIP/TH
Throughput	TH = WIP/CT
Bottleneck Rate	rb = 1/Max Avg Processing Time
Raw Processing Time	To = Sum of Avg Processing Time
Critical WIP	Wo = rb*To

BEST CASE PERFORMANCE

CT BEST	if $w \leq Wo$	To
	otherwise	W/rb
TH BEST	if $w \leq Wo$	w/To
	otherwise	rb

WORST CASE PERFORMANCE

CTworst	$= w*To$
THworst	$1/To$

PRACTICAL WORST CASE

CTpwc	$To + ((w-1)/rb)$
THpwc	$(w/(Wo+w-1))rb$

Sample Midterm Qsle Midterm Qs (Cont)

b) A company supplying seats to an auto assembly plant sends trucks to its customer at an average rate of 6 trucks per day. Given the travel time to the customer is an average of three days, what is the average number of trucks in transit at any given time?

TH = 6 trucks/day
CT = 3 days
WIP = TH x CT = 18 trucks

PREEMPTIVE ONLY

Natural Proc. Time	to
STD of Nat. Proc. Time	σ_o
SCV of Nat. Proc. Time	co^2 σ_o^2/to^2
STD of Nat. Proc. Time	mf
Mean Time to Repair	mr
STD of Time to Repair	σ_r
Mean Availability	A = $mf/(mf+mr)$
SCV of Time to Repair	cr^2 σ_r^2/mr
Mean Eff. Time with Preemptive Outages	$te(Po)$ to/A
SCV of Eff. Time with Preemptive Outages	$ce(Po)^2$ $co^2 + (1+cr^2)A(1-A)*mr/to$

PREEMPTIVE PLUS NON PREEMPTIVE

Mean batch size	Ns
Mean batch size	ts
STD of Setup Time	σ_s
Mean Eff. Time with Preemptive Outages	te $te(Po)+ts/Ns$
std. dev. Squared of eff. Time	σ_e^2 $te(po)^2 \times cd(po)^2 + (rs^2/Ns) + (Ns-1/Ns^2) \times ts^2$
SCV of Eff. Time with Preemptive Outages	ce^2 σ_e^2/te^2
Mean Utilization	u te/ta
SCV of interarrival times	ca^2
SCV of interdeparture times	cd^2 $1 + (1-u^2) \times (ca^2-1) + u^2(ce^2-1)$

PREEMPTIVE PLUS NON PREEMPTIVE (cont)

CTQ	$(ca^2 + ce^2)/2 \times u/(1-u) \times te$
CT	CTQ + te
WIP	CT/ta
WIP	CT/ta

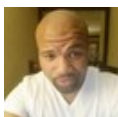
Sample Midterm Qsle Midterm Qs (Cont)

b) A company supplying seats to an auto assembly plant sends trucks to its customer at an average rate of 6 trucks per day. Given the travel time to the customer is an average of three days, what is the average number of trucks in transit at any given time?

TH = 6 trucks/day
CT = 3 days
WIP = TH x CT = 18 trucks

Sample Midterm Qs

SCV of Effective processing time	$ce^2 = ((\sigma_o^2 + (\sigma_s^2/N) + (Ns-1)/Ns^2 \times ts^2)/te$
Utilization	$u = (te/ta) = (te(np)/ta)$
Utilization (coffee sho)	$u = ra/re=te/ta$
WIP (M/M/1)	$u/(1-u)$
WIP (M/M/1)	$u/(1-u)$
CT (M/M/1)	$te/(1-u)$



By musikdr
cheatography.com/musikdr/

Published 3rd October, 2019.
Last updated 3rd October, 2019.
Page 1 of 2.

Sponsored by [Readable.com](https://readable.com)
Measure your website readability!
<https://readable.com>

Sample Midterm Qs (Cont)

3. a) Compute the average cycle time at machine 1.

$$CTq1 = (Ca^2 + Ce^2)/2 \times (u/(1-u)) \times te$$

$$u = (te/ta)$$

b) Compute the mean and coefficient of variation of the time between departures from Machine 1.

$$ta(2) = td(1) = ta(1)$$

$$cd^2 = 1 + (1-u^2) \times (ca^2 - 1) + u^2(ce^2 - 1)$$

$$u2 = te2/ta2 = 18/22$$

c) Compute the average cycle time at machine 2

$$CT(2) = CTq(2) + te(2) \text{ (note use } u2 \text{ to calculate } CTq(2))$$

d) Calculate the total CT and WIP of the system (combining machine 1 and 2).

$$\text{Total CT} = CT(1) + CT(2)$$

From Little's Law

$$\text{Total WIP} = CT \times TH = CT/ta$$

e) Now suppose the line must produce both products in equal proportion, i.e., one unit of Product 1 for each unit of Product 2.

Estimate the bottleneck rate and raw process time of the line under this product mix.

Hint: Think about the what the average processing time will be at each machine.
Processing time at $(M1 + M2)/\# \text{ of machines}$
(2 calculations .. 1 for each product)
 $rb = 1/\text{largest processing}$
 $to = to1 + to2$ (answer from processing time)

Kendall Notation

M	Memoryless or exponential
D	Deterministic
G	General
G/G/1	not exponential, gives approx CT and CTq
M/M/1	exponential, infinite source population unlimited queue length

M/M/1 Queuing

$$WIP = u/(1-u)$$

$$CT = WIP/ra = te/(1-u)$$

$$CTq = CT - te = (u \times te)/(1-u)$$

$$WIPq = raCTq = u^2/(1-u)$$

G/G/1 QUE

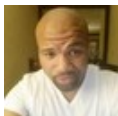
$$CT = ((ca^2 + ce^2)/2)(u/(1-u)) \times te$$

M/M/1/b

$$WIP = u/(1-u) - ((b+1)u^{b+1}) / (1-u^{b+1})$$

$$TH = ((1-u^b) / (1-u^{b+1})) \times ra$$

- Smaller buffer sizes bring greater losses relative to uncapacitated system



By musikdr
cheatography.com/musikdr/

Published 3rd October, 2019.
Last updated 3rd October, 2019.
Page 2 of 2.

Sponsored by **Readable.com**
Measure your website readability!
<https://readable.com>