Little's Law

WORK IN PROGRESS	WIP = TH*CT	
Cycle Time	CT	WIP/TH
Throughput	TH	WIP/CT
Bottleneck Rate	rb	=1/Max Avg Processing Time
Raw Processing Time	To	Sum of Avg Processing Time
Critical WIP	Wo	rb*To

BEST CASE PERFORMANCE

CT BEST	if w<=Wo	To
	otherwise	W/rb
TH BEST	if w<=Wo	w/To
	otherwise	rb

WORST CASE PERFORMANCE

CTworst	=w*To
THworst	1/To

PRACTICAL WORST CASE

CTpwc	To+((w-1)/rb)
THpwc	(w/(Wo+w-1))rb

Sample Midterm Qsle Midterm Qs (Cont)

b) A company supplying seats to an auto assembly plant sends trucks to its customer at an average rate of 6 trucks per day. Given the travel time to the customer is an average of three days, what is the average number of trucks in transit at any given time?

TH = 6 trucks/day

CT = 3 days

WIP = TH x CT = 18 trucks

PREEMPTIVE ONLY

Natural Proc. Time	to	
STD of Nat. Proc. Time	σo	
SCV of Nat. Proc. Time	co^2	$σo^2/to^2$
STD of Nat. Proc. Time	mf	
Mean Time to Repair	mr	
STD of Time to Repair	σr	
Mean Availability	A	mf/(mf+mr)
SCV of Time to Repair	cr^2	$σr^2/mr$
Mean Eff. Time with Preemptive Outages	te(Po)	to/A
SCV of Eff. Time with Preemptive Outages	$ce(PO)^2$	$co^2+(1+cr^2)A(1-A)*mr/to$

PREEMPTIVE PLUS NON PREEMPTIVE

Mean batch size	Ns	
Mean batch size	ts	
STD of Setup Time	σs	
Mean Eff. Time with Preemptive Outages	te	te(PO)+ts/Ns
std. dev. Squared of eff. Time	$σe^2$	$te(po)^2$ x $cd(po)^2+(rs^2/Ns)$ + $(Ns-1/Ns^2)$ x ts^2
SCV of Eff. Time with Preemptive Outages	ce^2	$σe^2/te^2$
Mean Utilization	u	te/ta
SCV of interarrival times	ca^2	
SCV of interdeparture times	cd^2	$1 + (1-u^2)$ x $(ca^2-1) + u^2(ce^2-1)$

PREEMPTIVE PLUS NON PREEMPTIVE (cont)

CTQ	$(ca^2 + ce^2)/2$ x u/(1-u) x te
CT	CTQ + te
WIP	CT/ta
WIP	CT/ta

Sample Midterm Qsle Midterm Qs (Cont)

b) A company supplying seats to an auto assembly plant sends trucks to its customer at an average rate of 6 trucks per day. Given the travel time to the customer is an average of three days, what is the average number of trucks in transit at any given time?

TH = 6 trucks/day

CT = 3 days

WIP = TH x CT = 18 trucks

Sample Midterm Qs

SCV of Effective processing time	$ce^2 = ((σo^2 + (σs^2/N) + (Ns-1)/Ns^2$ x $ts^2)/te$
Utilization	u = (te/ta) = (te(np)/ta)
Utilization (coffee sho)	u = ra/re=te/ta
WIP (M/M/1)	u/(1-u)
WIP (M/M/1)	u/(1-u)
CT (M/M/1)	te/(1-u)

Sample Midterm Qs (Cont)

3. a) Compute the average cycle time at machine 1.

CTq1 =(Ca^2 + Ce^2)/2 x (u/(1-u)) x te

u = (te/ta)

b) Compute the mean and coefficient of variation of the time between departures from Machine 1.

ta(2) = td(1) =ta(1)

cd^2 = 1 + (1-u^2) x (ca^2 -1) + u^2(ce^2 - 1)

u2 = te2/ta2 = 18/22

c) Compute the average cycle time at machine 2

CT(2) = CTq(2) + te(2) (note use u2 to calculate CTq(2)

d) Calculate the total CT and WIP of the system (combining machine 1 and 2).

Total CT = CT(1) + CT(2)

From Little's Law

Total WIP = CT x TH = CT/ta

e) Now suppose the line must produce both products in equal proportion, i.e., one unit of Product 1 for each unit of Product 2. Estimate the bottleneck rate and raw process time of the line under this product mix.

Hint: Think about the what the average processing time will be at each machine.

Processing time at (M1 + M2)/#of machines (2 calculations ..1 for each product)

rb = 1/largest processing

to = to1+to2 (answer from processing time)

Kendall Notation

M	Memoryless or exponential
D	Deterministic
G	General
G/G/1	not exponential, gives approx CT and CTq
M/M/1	exponential, infinite source population unlimited queue length

M/M/1 Queing

WIP = u/(1-u)

CT = WIP/ra = te/(1-u)

CTq = CT-te =(u x te)/(1-u)

WIPq =raCTq=u^2/(1-u)

G/G/1 QUE

CT = ((ca^2 + ce^2)/2)(u/(1-u) x te

M/M/1/b

WIP = u/(1-u) - ((b+1)u^{b+1}) / 1-u^{b+1}

TH = ((1-u^b) / (1-u^{b+1}) x ra

• Smaller buffer sizes bring greater losses relative to uncapacitated system