

Introduction

Statistical hypothesis	Statistical hypothesis testing
a hypothesis that is testable on the basis of observed data modeled as the realized values taken by a collection of random variables	a statistical way of testing the assumption regarding a popular parameter

steps of formulating a hypothesis

1. state the two hypothesis: **Null hypothesis** and **Alternative hypothesis**
2. set the **significance levels** usually $\alpha = 0.05$
3. carrying out the hypothesis testing and calculate the test statistics and corresponding **P-value**
4. compare P-value with significance levels and then decide to accept or reject null hypothesis

Errors in Testing

Error Types	Description	denotation	correct inference
Type I error	Reject null when null is true	$\alpha = P(\text{Type I error})$	$1 - \alpha$ (significance level)
Type II error	Not reject null when null is false	$\beta = P(\text{Type II error})$	$1 - \beta$ (= power)

Chi-Square Test

Types	Description
Test for independence	tests for the independence of two categorical variables
Homogeneity of Variance	test if more than two subgroups of a population share the same multivariate distribution
goodness of fit	whether a multinomial model for the population distribution (P_1, \dots, P_m) fits our data

Test for independence and homogeneity of variance share the same test statistics and degree of freedoms by different design of experiment

Assumptions

1. one or two categorical variables
2. independent observations
3. outcomes mutually exclusive
4. large n and no more than 20% of expected counts < 5

F-test

Anova Analysis	comparing the means of two or more continuous populations
One-way layout	A test that allows one to make comparisons between the means of two or more groups of data.
two-way layout	A test that allows one to make comparisons between the means of two or more groups of data, where two independent variables are considered.

Assumptions about data:

1. each data y is normally distributed
2. the variance of each treatment group is same
3. all observations are independent

T-test

Types	Hypothesis
Two Sample T-test	If two independent groups have different mean
Paired T-test	if one groups have different means at different times
One Sample T-test	mean of a single group against a known mean

Assumptions about data

1. independent
2. normally distributed
3. have a similar amount of variance within each group being compared



One sample T-test

$$t = \frac{m - \mu}{s/\sqrt{n}}$$

where

m = the mean of sample

s = standard deviation of sample

degree of freedom = n - 1

Paired T-test statistics

$$t = \frac{m}{s/\sqrt{n}}$$

where

m = the mean of differences between two paired sets of data

n = size of differences

s = the standard deviation of differences between two paired sets of data

degree of freedom = n - 1

Independent two-sample T-test statistics

$$t = \frac{m_A - m_B}{\sqrt{\frac{s^2}{n_A} + \frac{s^2}{n_B}}}$$

where

m = the means of group A and B respectively

n = the sizes of group A and B respectively

degrees of freedom = nA + nB - 2 (given two samples have the same variance)

Test of independence and Homogeneity of variance

$$\chi^2 = \sum [(O_{r,c} - E_{r,c})^2 / E_{r,c}]$$

where

$E_{r,c} = (N_r * N_c)/n$

df = (r - 1) * (c - 1)

c = column number

r = row number

Goodness of fit test

$$\chi_c^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

where:

O = observed value of data

E = expected value of data

k = dimension of parameter

df = n - 1 - k

Carrying out one-way anova test

SST total variance sum(Y_{ij} - overall mean of Y)²

SSW intra-group variance sum(mean of each observations across different treatments - mean of each treatment)²

SSB inter-group variance sum(mean of each treatments - overall mean of Y)²

Null hypothesis: the differentiated effect in each treatment group is 0

Alternative hypothesis: not all differentiated effect is 0

SST = SSW + SSB

test statistics:

$F_{i-1, i(j-1)} = SSB/(I-1)/SSW/I(J-1)$

where

I = number of different treatments

J = number of observations within each treatment