

DEFINITIONS

Even Integer An integer x is even if there is an integer k such that $x = 2k$.

Odd Integer An integer x is odd if there is an integer k such that $x = 2k+1$.

Parity Whether the number is odd or even

Divides An integer x divides an integer y if and only if $x \neq 0$ and $y = kx$, for some integer k .
Denoted $x|y$. If x does not divide y , then that fact is denoted $x \nmid y$. If x divides y , then y is said to be a **multiple** of x , and x is a **factor** or **divisor** of y .

Prime An integer n is prime if and only if $n > 1$, and the only positive integers that divide n are 1 and n .

Composite An integer n is composite if and only if $n > 1$, and there is an integer m such that $1 < m < n$ and m divides n .

Rational A number r is rational if there exist integers x and y such that $y \neq 0$ and $r = x/y$.

ZERO 0 is rational. For example if $x = 0$ and $y = 1$, then $y \neq 0$ and $x/y = 0/1 = 0$.

METHOD DEFINITIONS

constructive proof of existence

A proof that shows that an existential statement is true.

proof by exhaustion

Allowed assumptions in proofs

The rules of algebra.

For example if x , y , and z are real numbers and $x = y$, then $x+z = y+z$.

The set of integers is closed under addition, multiplication, and subtraction.

In other words, sums, products, and differences of integers are also integers.

Every integer is either even or odd.

This fact is proven elsewhere in the material.

If x is an integer, there is no integer between x and $x+1$.

In particular, there is no integer between 0 and 1.

The relative order of any two real numbers.

For example $1/2 < 1$ or $4.2 \geq 3.7$.

The square of any real number is greater than or equal to 0.

This fact is proven in a later exercise.

Common keywords and phrases in proofs

Thus, therefore then, hence, it follows that
A statement that follows from the previous statement(s)

ex. n and m are integers. Therefore, $n+m$ is also an integer.

Let, suppose

Introduce a new variable

ex. "Let x be a positive integer" "Suppose that x is a positive integer"

Since

If a statement depends on a fact that appeared earlier in the proof or in the assumptions of the theorem, it can be helpful to remind the reader of that fact before the statement.

ex. "Since $x > 0$ and $y > z$, then $xy > xz$."

By definition

A fact that is known because of a definition
ex. "The integer m is even. By definition, $m = 2k$ for some integer k ."

By assumption

A fact that is known because of an assumption

ex. "By assumption, x is positive. Therefore $x > 0$."

"gives" and "yields"

useful to say that one equation or inequality follows from another

provides clarity to justify algebraic steps
**ex. Multiplying both sides of the inequality $x > y$ by 2 gives $2x > 2y$.*

Substituting $m = 2k$ into m^2 yields $(2k)^2$

Since $z > 0$, we can multiply both sides of the inequality $x > y$ by z to get $xz > yz$.

Choosing a Method

