DEFINITIONS

Even Integer	An integer x is even if there is an integer k such that x = 2k.
Odd Integer	An integer x is odd if there is an integer k such that x = 2k+1.
Parity	Whether the number is odd or even
Divides	An integer x divides an integer y if and only if x ≠ 0 and y = kx, for some integer k. Denoted x|y. If x does not divide y, then that fact is denoted x ∤ y. If x divides y, then y is said to be a **multiple** of x, and x is a **factor** or **divisor** of y.
Prime	An integer n is prime if and only if n > 1, and the only positive integers that divide n are 1 and n.
Composite	An integer n is composite if and only if n > 1, and there is an integer m such that 1 < m < n and m divides n.
Rational	A number r is rational if there exist integers x and y such that y ≠ 0 and r = x/y.
ZERO	0 is rational. For example if x = 0 and y = 1, then y ≠ 0 and x/y = 0/1 = 0.

METHOD DEFINITIONS

constructive proof of existence

> A proof that shows that an existential statement is true.

proof by exhaustion

Allowed assumptions in proofs

The rules of algebra.

> For example if x, y, and z are real numbers and x = y, then x+z = y+z.

The set of integers is closed under addition, multiplication, and subtraction.

> n other words, sums, products, and differences of integers are also integers.

Every integer is either even or odd.

> This fact is proven elsewhere in the material.

If x is an integer, there is no integer between x and x+1.

> In particular, there is no integer between 0 and 1.

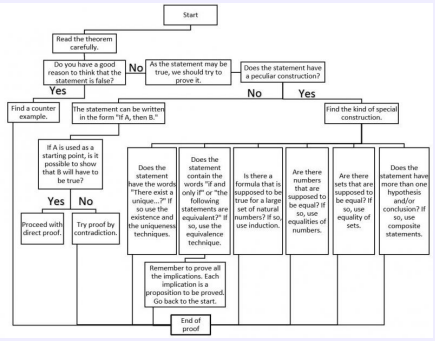
The relative order of any two real numbers.

> For example 1/2 < 1 or 4.2 ≥ 3.7.

The square of any real number is greater than or equal to 0.

> This fact is proven in a later exercise.

Choosing a Method



Common keywords and phrases in proofs

Thus, therefore then, hence, it follows that

A statement that follows from the previous statement(s)

ex. n and m are integers. Therefore, n+m is also an integer.

Let, suppose

Introduce a new variable

ex. "Let x be a positive integer" "Suppose that x is a positive integer"

Since

If a statement depends on a fact that appeared earlier in the proof or in the assumptions of the theorem, it can be helpful to remind the reader of that fact before the statement.

ex. "Since x > 0 and y > z, then xy > xz."

By definition

A fact that is known because of a definition

ex. "The integer m is even. By definition, m = 2k for some integer k."

By assumption

A fact that is known because of an assumption

ex. "By assumption, x is positive. Therefore x > 0."

"gives" and "yields"

useful to say that one equation or inequality follows from another

provides clarity to justify algebraic steps

*ex. Multiplying both sides of the inequality x > y by 2 gives 2x > 2y.

Substituting m = 2k into m2 yields (2k)2*

Since z > 0, we can multiply both sides of the inequality x > y by z to get xz > yz.