SECTION 1

C, Fields, Lists, F^n, Vector Spaces

Definition of complex numbers (n∈C)

C = { a+bi : a,b ∈ *R* }	
addition in C (a+bi) + (c+di) = (a+c) + (b+d)i	
multiplication in C (a+bi)(c+di) = (ac-bd) + (ad+bc)i	

Properties of Fields

commutativity	a+b = b+a ∧ ab = ba ∀a,b ∈ **F**
associativity	(a+b)+c = a+(b+c) ∧ (ab)c = a(bc) ∀a,b,c ∈ **F**
identities	c+0 = c ∧ 1c = c ∀c ∈ **F**
additive inverse	∀a ∈ **F** ∃b ∈ **F** : a+b = 0
multiplicative inverse	a ∈ **F**, a ≠ 0, ∃b ∈ **F** : a+b = 0
distributive property	c(a+b) = ca + cb ∀a,b,c ∈ **F**

Definition of list L with length n

n∈***N***, **L** : { 1,2,…,n } → Elements

L = (x_1,...,x_n)

Definition of Fn

F^n = { (x_1,...,x_n) : x_j ∈ *F* for j = 1,...,*n*}	
addition in F^n	(x_1,...,x_n) + (y_1,...,y_n) = (x_1+y_1,...,x_n+y_n)
zero vector	*0* = (0,...,0)
additive inverse	For *x* ∈ F^n, -*x* ∈ F^n = (-x_1,...,-x_n)
scalar multiplication in F^n	c ∈ *F*, c(x_1,...,x_n) = (cx_1,...,cx_n)

Definition of addition and scalar multiplication

addition on a set V +	: (*u*,*v* ∈ *V*) → *u* +*v*
scalar multiplication on a set V *	: (c ∈ *F*, *v* ∈ *V*) → c *v*

Defintion of Vector Space

Vector Space = *V* + (*+*) + (*)	
commutativity	*u*+*v* = *v*+*u*
associativity	(*u*+*v*)+*w* = *u*+(*v*+*w*) ∧ (ab) *v* = a(b *v*)
additive identity	∃ *0* ∈ *V* : *v*+*0* = *v*
additive inverse	∀ *v* ∈ *V*, ∃ *w* ∈ *V* : *v*+*w* = *0*
multiplicative identity	1 *v* = *v*
distributive property	a(*v*+*w*) = a*v* + a*w* ∀a ∈ **F**
v ∈ *V* = vector v point	