SECTION 1

C, Fields, Lists, F^n, Vector Spaces

Definition of complex numbers (n∈C)

$C = \{ a+bi : a,b \in R \}$
addition in C (a+bi) + (c+di) = (a+c) + (b+d)i
multiplication in C (a+bi)(c+di) = (ac-bd) + (ad+bc)i

Properties of Fields

commutativity a+b = b+a ∧ ab = ba ∀a,b ∈ **F**
associativity (a+b)+c = a+(b+c) ∧ (ab)c = a(bc) ∀a,b,c ∈ **F**
identities c+0 = c ∧ 1c = c ∀c ∈ **F**
additive inverse ∀a ∈ **F** ∃b ∈ **F** : a+b = 0
multiplicative inverse a ∈ **F**, a ≠ 0, ∃b ∈ **F** : a+b = 0
distributive property c(a+b) = ca + cb ∀a,b,c ∈ **F**

Definition of list L with length n

n∈***N***, **L** : { 1,2,…,n } → Elements
L = $(x_1,...,x_n)$

Definition of Fn

$F^n = \{ (x_1,...,x_n) : x_j \in F$ for $j = 1,...,n\}$
addition in F^n $(x_1,...,x_n) + (y_1,...,y_n) = (x_1+y_1,...,x_n+y_n)$
zero vector $0 = (0,...,0)$
additive inverse For $x \in F^n$, $-x \in F^n = (-x_1,...,-x_n)$
scalar multiplication in F^n $c \in F$, $c(x_1,...,x_n) = (cx_1,...,cx_n)$

Definition of addition and scalar multiplication

addition on a set V + : $(u,v \in V) \to u+v$
scalar multiplication on a set V * : $(c \in F, v \in V) \to cv$

Defintion of Vector Space

Vector Space = V + (+) + (*)
commutativity $u+v = v+u$
associativity $(u+v)+w = u+(v+w) \wedge (ab)v = a(bv)$
additive identity $\exists 0 \in V: v+0 = v$
additive inverse $\forall v \in V, \exists w \in V: v+w = 0$
multiplicative identity $1v = v$
distributive property $a(v+w) = av + aw$ ∀a ∈ **F**
$v \in V$ = vector v point