## SECTION 1

*C*, Fields, Lists, $F^n$, Vector Spaces

### Definition of complex numbers (n∈C)

*C* = { a+bi : a,b ∈ *R* }

***addition in C*** (a+bi) + (c+di) = (a+c) + (b+d)i

***multiplication in C*** (a+bi)(c+di) = (ac-bd) + (ad+bc)i

### Properties of Fields

**commutativity** a+b = b+a ∧ ab = ba ∀a,b ∈ **F**

**associativity** (a+b)+c = a+(b+c) ∧ (ab)c = a(bc) ∀a,b,c ∈ **F**

**identities** c+0 = c ∧ 1c = c ∀c ∈ **F**

**additive inverse** ∀a ∈ **F** ∃b ∈ **F** : a+b = 0

**multiplicative inverse** a ∈ **F**, a ≠ 0, ∃b ∈ **F** : a+b = 0

**distributive property** c(a+b) = ca + cb ∀a,b,c ∈ **F**

### Definition of list L with length n

n∈***N***, **L** : { 1,2,…,n } → Elements

**L** = $(x_1,\dots,x_n)$

### Definition of Fn

$F^n = \{ (x_1,\dots,x_n) : x_j \in F \text{ for } j = 1,\dots,n \}$

***addition in*** $F^n$ $(x_1,\dots,x_n) + (y_1,\dots,y_n) = (x_1+y_1,\dots,x_n+y_n)$

***zero vector*** *0* = (0,...,0)

***additive inverse*** For $x \in F^n$, $-x \in F^n = (-x_1,\dots,-x_n)$

***scalar multiplication in*** $F^n$ c ∈ *F*, $c(x_1,\dots,x_n) = (cx_1,\dots,cx_n)$

### Definition of addition and scalar multiplication

***addition on a set V +***: (*u*,*v* ∈ *V*) → *u* +*v*

***scalar multiplication on a set V ****: (c ∈ *F*, *v* ∈ *V*) → c *v*

### Defintion of Vector Space

Vector Space = *V* + (*+*) + (***)

**commutativity** *u*+*v* = *v*+*u*

**associativity** (*u*+*v*)+*w* = *u*+(*v*+*w*) ∧ (ab) *v* = a(b *v*)

**additive identity** ∃ *0* ∈ *V* : *v*+*0* = *v*

**additive inverse** ∀ *v* ∈ *V*, ∃ *w* ∈ *V* : *v*+*w* = *0*

**multiplicative identity** 1 *v* = *v*

**distributive property** a(*v*+*w*) = a*v* + a*w* ∀a ∈ **F**

*v* ∈ *V* = vector v point