

# Probability - Midterm Cheat Sheet by madsysharma via cheatography.com/208834/cs/44841/

Special Distributions (Discrete RVs)			
E and Var	NAME	RX	PMF
p & p(1-p)	Bernou- lli(p)	{0,1}	p for x=1, 1-p for x=0
1/p and (1- p)/p <sup>2</sup>	Geomet ric(p)	Z <sup>+</sup>	$p(1-p)^{(k-1)}$ for $k \in \mathbb{Z}^+$
np and np(1-p)	Binomi- al(n,p)	{0,1,,n}	${}^{n}C_{k} \cdot p^{k} \cdot (1-p)^{(n-k)}$ for $k = 0$ to
m/p and (m.(1 p))/p <sup>2</sup>	Pascal- (m,p)	{m,m+1, m+2}	$^{(k-1)}C(m-1)$ $p^m \cdot (1-p)^{(k-m)}$ for $k = m, m+1, m+2, m+3,$
np and ((b+r n)/(b+r- 1)).n- p(1-p)	Hyperg- eometr- ic(b,r,k)	{max(0,k -r), max(0,k- r)+1,, min(k,b)}	$({}^{b}C_{X} \cdot {}^{r}C_{(k-x)})/({}^{(b+r)}C_{k})$ $\forall x \in R_{X}$
Both equal to lambda	Poisso- n(l- ambda)	Z <sup>+</sup>	(e <sup>-lambda</sup> . lambda <sup>k</sup> )/k! for k <del>C</del> R <sub>X</sub>

### Continuous RVs, PDFs and Mixed RVs

RV X with CDF  $F_X(x)$  is continuous if  $F_X(x)$  is a continuous function  $\forall x \in R$  PMF doesn't work for CRVs, since  $\forall x \in R$ ,  $P_X(x) = 0$ . Instead, PDFs are used. PDF =  $f_X(x) = dF_X(x)/dx$  (if  $F_X(x)$  is differentiable at x) >=0  $\forall x \in R$ . P(a<X<=b) = integral from a to b ( $f_X(u)$ .du) and integral from -inf to +inf ( $f_X(u)$ .du) = 1

# Continuous RVs, PDFs and Mixed RVs (cont)

EX = integral from -inf to +inf  $(x \cdot f_X(x) \cdot dx)$ 

and E[g(X)] = integral from -inf to +inf (g(x) .  $f_X(x)$  . dx)

Var(X) = integral from -inf to +inf ( $x^2$  .  $f_X(x)$  . dx -  $mu^2_X$ )

If g: R-> R is strictly monotonic and differentiable, then PDF of Y=g(X) is  $f_Y(y)$  =  $f_X(x_1)$  .  $|dx_1/dy|$  where  $g(x_1)$ =y and 0 if g(x)

### Joint Distributions: RVs >= 2

= y has no solution

Joint PMF of X and Y =  $P_{XY}(x,y) = P(X=x,y)$ Y=y) = P((X=x) and (Y=y)) and Joint range =  $R_{XY} = \{(x,y)| P_{XY}(x,y) > 0\}$  and summing up PXY over all (x,y) pairs will result in 1 Marginal PMF of  $X = P_X(x) = \text{sum over all } y_i$  $GR_Y(P_{XY}(x, y_i))$  for any  $x GR_X$ . Similarly, Marginal PMF of Y =  $P_Y(y)$  = sum over all  $x_i$  $G_{XY}(P_{XY}(x_i, y))$  for any  $y G_{Y}$ To show independence between X and Y, prove  $P(X = x, Y=y) = P(X=x) \cdot P(Y=y)$  for all x-y pairs. Similarly, for conditional independence, show that P(Y=y|X=x) = P(Y=y) for all x-y pairs Joint CDF =  $F_{XY}(x,y) = P(X \le x, Y \le y)$  and Marginal CDF for  $X = F_X(x) = limit y to$  $\inf(F_{XY}(x,y))$  for any x and Marginal CDF for  $Y = F_Y(y) = \text{limit } x \text{ to inf } (F_{XY}(x,y)) \text{ for }$ any y Conditional expectation:  $E[X|Y=y_i] = sum$ over all  $x_i \in R_X(x_i . P_{X|Y}(x_i|y_i))$ NOTE:  $F_{XY}(inf,inf) = 1$ ,  $F_{XY}(-inf,y) = 0$  for any y and  $F_{XY}(x,-inf) = 0$  for any x  $P(x_1 < X \le x_2, y_1 < Y \le y_2) = F_{XY}(x_1, y_1)$  $+ F_{XY}(x_2,y_2) - F_{XY}(x_2,y_1) - F_{XY}(x_1,y_2)$ Conditional PMF given event  $A = P_{X|A}(x_i) =$  $P(X=x_i|A) = P(X=x_i \text{ and } A)/P(A) \text{ for any } x_i \in$  $R_X$  and Conditional CDF =  $F_{X|A}(x)$  = P(X)<= x | A)

### Joint Distributions: RVs >= 2 (cont)

Given RVs X and Y,  $P_{X|Y}(x_i, y_j) = P_{XY}(x_i, y_j)/P_Y(y_j)$ . Similarly for Y|X E[X + Y] = E[X] + E[Y] - independence not required E[X . Y] = E[X] . E[Y] - independence IS required

# Problem Solving Techniques

#### \* CARD PROBLEMS:

Number of ways to pick k suits =  ${}^{4}C_{k}$  with k=1,2,3,4

#### \* n BALLS, r BINS:

- Distinguishable balls: each ball can go into any 1 of r bins. The # of distinct perms would be  ${}^{r}P_{n} = r^{n}$
- Indistinguishable balls: there will be 2 cases:
- \* No empty bins. Occupancy vector is  $x_1+...+x_r=n$  where every x is >= 1. There can be n-1 possible locations for bin dividers from which we can choose r-1 to keep >= 1 ball in each bin. # of possible arrangements =  $\binom{(n-1)}{(r-1)}$ .
- \* Bin may have 0 balls. Then the occupancy vector would be  $y_1+...+y_r=n+r$  and the # of arrangements will be  ${}^{(n+r-1)}C_{(r-1)}$
- \* COMMITTEE SELECTION: Solve using product rule/hypergeometric approach.

### \* HAT MATCHING PROBLEM:

➤ Probability of k men drawing their own hats (over all k-tuples) = (<sup>n</sup>C<sub>k</sub>(n-k)!)/n! = 1/k! # of derangements = n![1-1/1!+1/2!--1/3!+...+(-1)<sup>n</sup>/n!]

> P(k matches) = [1/2! - 1/3! + 1/4! -...+(-1)<sup>(n-k)</sup>/(n-k)!]/k!

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### **Problem Solving Techniques (cont)**

## \* DRAWING THE ONLY SPECIAL BALL FROM n BALLS IN k TRIALS:

Total # of outcomes =  ${}^{n}C_{k} = {}^{[1 + (n-1)]}C_{k} = {}^{1}C_{0}{}^{(n-1)}C_{k} + {}^{1}C_{1}{}^{(n-1)}C_{(k-1)}$ , with term #1 denoting no special ball, and term #2 denoting the special ball

\*Total # of roundtable arrangements with k people = k!/k = (k-1)!

#### \* SYSTEM RELIABILITY ANALYSIS:

- > P(fail)=p, P(success)=1-p
- ➤ For parallel config, 2<sup>n</sup>-1 successes and 1 failure, P(fail)=p<sup>n</sup>
- ➤ For series config, 2<sup>n</sup>-1 failures and 1 success, P(success) = (1-p)<sup>n</sup>
- ➤ For series connections, take intersection, and for parallel connections take union
- \* PMF FOR SUM, DIFF, MAX, MIN OF 4-SIDED DICE:
- $\blacktriangleright$  Uniform PMF =  $P_{XY}(x,y) = 1/16$
- ➤ For each (x,y) point in the Cartesian coordinate diagram, calculate the diff/sum label or min/max label.
- ➤ Write down tables for Joint, Marginal and Conditional PMFs
- ▶ Headers are:  $x y P_{XY}(x,y) x P_{X}(x) y$  $P_{Y}(y) x y P_{Y|X}(y,x)$ . First 3 for joint, next 4 for marginal, the remaining for conditional
- ➤ For marginal, plot PMF on y-axis and RV value on x-axis.
- > For joint, plot y on y-axis and x on x-axis

### Facts for PMFs and RV Distributions

 $0 \le P_X(x) \le 1 \ \forall \ x \text{ and Sum over all } x \in R_X$  $(P_X(x)) = 1$ 

For any set  $A \subseteq R_X$ ,  $P(X \subseteq A) = \sum_{X \subseteq A} P_X(x)$ RVs X and Y are independent if P(X = x, Y = y) = P(X = x) \* P(Y = y),  $\forall x, y$  The first formula can be extended to n times. P(Y = y | X = x) = P(Y = y),  $\forall x, y$  if X & Y are independent

If  $X_1,...,X_n$  are independent Bernoulli(p) RVs, then  $X=X_1+X_2+...+X_n$  has Binomial(n,p) distribution, and **Pascal (1,p) = Geometric (p)** 

TYPE, PDF & E[X] AND VAR(X) Uniform(a, b) || 1/(b-a) if a<x<br/>b || (a+b)/2 and (b-a) $^2$ /12

$$\begin{split} & \text{Exponential(lambda)} \parallel \text{lambda} \cdot e^{(-\text{lambda} \cdot x)} \\ \parallel 1/\text{lambda} \text{ and } 1/(\text{lambda})^2 \\ & \text{Normal/Gaussian, ie: N(0,1)} \parallel (1/\text{sqrt}(2 \cdot pi)) \cdot \exp(-x^2/2), \ \forall \ x \in R \parallel 0 \ \text{and } 1 \\ & \text{Gamma (alpha, lambda)} \parallel (\text{lambda}^{\text{alpha}} \cdot x^{(\text{alpha}-1)} \cdot e^{(-\text{lambda} \cdot x)})/(\text{alpha}-1)! \ \text{for } x > 0 \parallel \text{alpha/lambda} \ \text{and } EX/\text{lambda} \end{split}$$

CDF:  $F_X(x) = P(X \le x) \forall x \in R$  and  $P(a \le X \le b) = F_X(b) - F_X(a)$ 

### Counting Principles, n-nomial Expansions

Permutations of n distinct objs. take n w/ r groups of indistinct objs. =  $(n!)/(n_1! ... n_r!)$  ${}^{n}P_{r} = {}^{r}$  and  ${}^{n}C_{r} = {}^{(n+r-1)}C_{r}$ : for perms and combs where k objs are taken at a time  $(a+b)^n$  = Sum over k ( $^nC_ka^kb^{(n-k)}$ ) where k=0,...,n Binomial coeff. identity:  ${}^{n}C_{k} = {}^{(n-1)}C_{(k-1)} +$  $^{(n-1)}C_k$  where first term maps to A and second to AC  ${}^{n}C_{m} = {}^{n}C_{n-m}$ Sum over  $r(^{n}C_{r}(-1)^{r}(1)^{(n+r)})$  is 0 where r=0,...n Sum over r  $((^{n}C_{r})^{2}) = {}^{2n}C_{n}$  where r=0,...,n Sum over s ( ${}^{s}C_{m}$ ) =  ${}^{n}(n+1)C_{m}$ (m+1) where s=m,..,n Hypergeometric expansion:  $^{(n+m)}C_r =$  ${}^{n}C_{0}{}^{m}C_{r} + {}^{n}C_{1}{}^{m}C_{(r-1)} + ... + {}^{n}C_{r}{}^{m}C_{0}$  a CE and ME enumeration  $n! = (n/e)^n x root(2n x pi) - Stirling's approx.$ Trinomial expansion: (a+b+c)<sup>n</sup> = sum over i, j, k (C'a<sup>i</sup>b<sup>j</sup>c<sup>k</sup>) where i, j, k=0,...n and i+j+k=n and C'=n!/(i!j!k!) In n-nomial expansion  $(a_1 + ... + a_r)^n$ , the #

# Expectation, Variance, RV Functions

1)C(r-1)

Expected value of X, ie:  $EX/E[X]/mu_X =$ sum over all  $x_k \in R_X (x_k \cdot P(X = x_k))$ . It is linear  $E[aX + b] = aE[X] + b, \forall a,b \in R$ 

of terms in the sum is  ${}^{r}C_{n} = {}^{(r+n-1)}C_{n} = {}^{(r+n-1)}C_{n}$ 

 $E[X_1 + ... + X_n] = E[X_1] + ... + E[X_n]$ 

If X is an RV and Y=g(X), then Y is also an RV.



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### Expectation, Variance, RV Functions (cont)

 $R_Y = \{g(x) \mid x \in R_X\}$  and  $P_Y(y) = sum over$ 

all x:g(x)=y ( $P_X(x)$ )

 $E[g(X)] = sum over all x_k \in R_X (g(x_k))$ .

 $P_X(x_k)$ ) (LOTUS)

 $Var(X) = E[(X - mu_X)^2] = sum over all x_k \in$ 

 $R_X ((x_k - mu_X)^2.P_X(x_k))$ 

 $SD(X)/sigma_X = sqrt(Var(X))$ 

Covariance = Cov(X,Y) = E[XY] - E[X].E[Y],

which will be 0 if X & Y are independent

 $Var(X) = Cov(X,X) = E[X^2] - (E[X])^2$ 

 $Var(aX + b) = a^2 Var(X)$ , and if  $X = X_1 + ... +$ 

 $X_n$ , then  $Var(X) = Var(X_1) + ... + Var(X_n)$ 

 $Var(aX + bY) = a^{2}Var(X) + b^{2}Var(Y) +$ 

2abCov(X,Y)

 $Var(total up to X_n) = sum of all <math>Var(X_i)$  if  $X_i$ 

is mutually independent for i = 1...n.

Summing up over the same conditions for expected values holds true, regardless of

independence or not

Correlation coefficient = Cov(X,Y)/(SD(X)).

SD(Y)) - ranges between -1 and 1 (inclusive

for both limits)

Z-standardized transformation: Z=(X -

 $mu_X$ )/SD(X) - zero mean and unit variance



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