

Special Distributions (Discrete RVs)

E and Var	NAME	R_X	PMF
p & $p(1-p)$	Bernoulli(p)	$\{0, 1\}$	p for $x=1$, $1-p$ for $x=0$
$1/p$ and $(1-p)/p^2$	Geometric(p)	Z^+	$p(1-p)^{(k-1)}$ for $k \in Z^+$
np and $np(1-p)$	Binomial(n,p)	$\{0, 1, \dots, n\}$	${}^n C_k \cdot p^k \cdot (1-p)^{(n-k)}$ for $k = 0$ to n
m/p and $(m \cdot (1-p)) / p^2$	Pascal(m,p)	$\{m, m+1, m+2, \dots\}$	${}^{(k-1)} C_{(m-1)} \cdot p^m \cdot (1-p)^{(k-m)}$ for $k = m, m+1, m+2, m+3, \dots$
np and $((b+r-n)/(b+r-1)) \cdot n \cdot p(1-p)$	Hypergeometric(b,r,k)	$\{\max(0, k-r), \dots, \min(k, b)\}$	${}^b C_k \cdot {}^r C_{(k-x)} / ({}^{(b+r)} C_k)$ $\forall x \in R_X$
Both equal to lambda	Poisson(lambda)	Z^+	$(e^{-\lambda} \cdot \lambda^k) / k!$ for $k \in R_X$

Continuous RVs, PDFs and Mixed RVs

RV X with CDF $F_X(x)$ is continuous if $F_X(x)$ is a continuous function $\forall x \in R$
 PMF doesn't work for CRVs, since $\forall x \in R$, $P_X(x) = 0$. Instead, PDFs are used.
 PDF = $f_X(x) = dF_X(x)/dx$ (if $F_X(x)$ is differentiable at x) $\geq 0 \forall x \in R$.
 $P(a < X \leq b) = \text{integral from } a \text{ to } b (f_X(u) \cdot du)$
 and $\text{integral from } -\text{inf to } +\text{inf} (f_X(u) \cdot du) = 1$

Continuous RVs, PDFs and Mixed RVs (cont)

$EX = \text{integral from } -\text{inf to } +\text{inf} (x \cdot f_X(x) \cdot dx)$
 and $E[g(X)] = \text{integral from } -\text{inf to } +\text{inf} (g(x) \cdot f_X(x) \cdot dx)$
 $\text{Var}(X) = \text{integral from } -\text{inf to } +\text{inf} (x^2 \cdot f_X(x) \cdot dx - \mu^2)$
 If $g: R \rightarrow R$ is strictly monotonic and differentiable, then PDF of $Y=g(X)$ is $f_Y(y) = f_X(x_1) \cdot |dx_1/dy|$ where $g(x_1)=y$ and 0 if $g(x) = y$ has no solution

Joint Distributions: RVs ≥ 2

Joint PMF of X and $Y = P_{XY}(x,y) = P(X=x, Y=y) = P((X=x) \text{ and } (Y=y))$ and **Joint range = $R_{XY} = \{(x,y) | P_{XY}(x,y) > 0\}$ and **summing up P_{XY} over all (x,y) pairs will result in 1**
Marginal PMF of $X = P_X(x) = \text{sum over all } y_j \in R_Y (P_{XY}(x, y_j))$ for any $x \in R_X$. Similarly, **Marginal PMF of $Y = P_Y(y) = \text{sum over all } x_i \in R_X (P_{XY}(x_i, y))$ for any $y \in R_Y$**
 To show independence between X and Y , **prove $P(X = x, Y=y) = P(X=x) \cdot P(Y=y)$ for all x - y pairs.** Similarly, for **conditional independence, show that $P(Y=y|X=x) = P(Y=y)$ for all x - y pairs**
Joint CDF = $F_{XY}(x,y) = P(X \leq x, Y \leq y)$ and Marginal CDF for $X = F_X(x) = \text{limit } y \text{ to } \text{inf}(F_{XY}(x,y))$ for any x and Marginal CDF for $Y = F_Y(y) = \text{limit } x \text{ to } \text{inf}(F_{XY}(x,y))$ for any y
Conditional expectation: $E[X|Y=y_j] = \text{sum over all } x_i \in R_X (x_i \cdot P_{X|Y}(x_i|y_j))$
NOTE: $F_{XY}(\text{inf}, \text{inf}) = 1$, $F_{XY}(-\text{inf}, y) = 0$ for any y and $F_{XY}(x, -\text{inf}) = 0$ for any x
 $P(x_1 < X \leq x_2, y_1 < Y \leq y_2) = F_{XY}(x_2, y_2) - F_{XY}(x_2, y_1) - F_{XY}(x_1, y_2) + F_{XY}(x_1, y_1)$
Conditional PMF given event $A = P_{X|A}(x_i) = P(X=x_i|A) = P(X=x_i \text{ and } A) / P(A)$ for any $x_i \in R_X$ and **Conditional CDF = $F_{X|A}(x) = P(X \leq x | A)$****

Joint Distributions: RVs ≥ 2 (cont)

Given RVs X and Y , $P_{X|Y}(x_i, y_j) = P_{XY}(x_i, y_j) / P_Y(y_j)$. Similarly for $Y|X$
 $E[X + Y] = E[X] + E[Y]$ - independence not required
 $E[X \cdot Y] = E[X] \cdot E[Y]$ - independence IS required

Problem Solving Techniques

- * **CARD PROBLEMS:**
 Number of ways to pick k suits = ${}^4 C_k$ with $k=1,2,3,4$
- * **n BALLS, r BINS:**
 - Distinguishable balls: each ball can go into any 1 of r bins. The # of distinct perms would be r^n
 - Indistinguishable balls: there will be 2 cases:
 * No empty bins. Occupancy vector is $x_1 + \dots + x_r = n$ where every $x_i \geq 1$. There can be $n-1$ possible locations for bin dividers from which we can choose $r-1$ to keep ≥ 1 ball in each bin. # of possible arrangements = ${}^{(n-1)} C_{(r-1)}$.
 * Bin may have 0 balls. Then the occupancy vector would be $y_1 + \dots + y_r = n+r$ and the # of arrangements will be ${}^{(n+r-1)} C_{(r-1)}$
- * **COMMITTEE SELECTION: Solve using product rule/hypergeometric approach.**
- * **HAT MATCHING PROBLEM:**
 > Probability of k men drawing their own hats (over all k -tuples) = $({}^n C_k (n-k)! / n!) = 1/k!$
 # of derangements = $n! [1 - 1/1! + 1/2! - 1/3! + \dots + (-1)^n / n!]$
 > $P(k \text{ matches}) = [1/2! - 1/3! + 1/4! - \dots + (-1)^{(n-k)} / (n-k)!] / k!$



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Problem Solving Techniques (cont)

* DRAWING THE ONLY SPECIAL BALL FROM n BALLS IN k TRIALS:

Total # of outcomes = ${}^n C_k = [1 + (n-1)] C_k = {}^1 C_0 (n-1) C_k + {}^1 C_1 (n-1) C_{(k-1)}$, with term #1 denoting no special ball, and term #2 denoting the special ball

*Total # of roundtable arrangements with k people = $k!/k = (k-1)!$

* SYSTEM RELIABILITY ANALYSIS:

- $P(\text{fail})=p$, $P(\text{success})=1-p$
- For parallel config, $2^n - 1$ successes and 1 failure, $P(\text{fail})=p^n$
- For series config, $2^n - 1$ failures and 1 success, $P(\text{success}) = (1-p)^n$
- For series connections, take intersection, and for parallel connections take union

* PMF FOR SUM, DIFF, MAX, MIN OF 4-SIDED DICE:

- Uniform PMF = $P_{XY}(x,y) = 1/16$
- For each (x,y) point in the Cartesian coordinate diagram, calculate the diff/sum label or min/max label.
- Write down tables for Joint, Marginal and Conditional PMFs
- Headers are: $x \ y \ P_{XY}(x,y) \ x \ P_X(x) \ y \ P_Y(y) \ x \ y \ P_{Y|X}(y,x)$. First 3 for joint, next 4 for marginal, the remaining for conditional
- For marginal, plot PMF on y-axis and RV value on x-axis.
- For joint, plot y on y-axis and x on x-axis

Facts for PMFs and RV Distributions

$0 \leq P_X(x) \leq 1 \ \forall x$ and **Sum over all $x \in R_X$**

$$(P_X(x)) = 1$$

For any set $A \subset R_X$, $P(X \in A) = \sum_{x \in A} P_X(x)$

RVs X and Y are independent if $P(X=x, Y=y) = P(X=x) * P(Y=y)$, $\forall x,y$. The first formula can be extended to n times.

$P(Y=y|X=x) = P(Y=y)$, $\forall x,y$ if X & Y are independent

If X_1, \dots, X_n are independent Bernoulli(p)

RVs, then $X=X_1+X_2+\dots+X_n$ has Binomial(n,p) distribution, and **Pascal (1,p) =**

Geometric (p)

For distributions using parameter p, $0 < p < 1$

If X is of Binomial (n, p = lambda/n), with fixed lambda > 0. Then, for any $k \in \mathbb{Z}$, $\lim_{n \rightarrow \infty} P_X(k) = (e^{-\text{lambda}} \cdot \text{lambda}^k) / k!$

* **SPECIAL DISTRIBUTIONS:**

TYPE, PDF & E[X] AND VAR(X)

Uniform(a, b) || $1/(b-a)$ if $a < x < b$ || $(a+b)/2$ and $(b-a)^2/12$

Exponential(lambda) || $\text{lambda} \cdot e^{-\text{lambda} \cdot x}$ || $1/\text{lambda}$ and $1/(\text{lambda})^2$

Normal/Gaussian, ie: $N(0,1)$ || $(1/\sqrt{2 \cdot \pi}) \cdot \exp(-x^2/2)$, $\forall x \in \mathbb{R}$ || 0 and 1

Gamma (alpha, lambda) || $(\text{lambda}^{\text{alpha}} \cdot x^{(\text{alpha}-1)} \cdot e^{-\text{lambda} \cdot x}) / (\text{alpha}-1)!$ for $x > 0$ || $\text{alpha}/\text{lambda}$ and $E[X]/\text{lambda}$

CDF: $F_X(x) = P(X \leq x) \ \forall x \in \mathbb{R}$ and $P(a <$

$X \leq b) = F_X(b) - F_X(a)$

Counting Principles, n-nomial Expansions

Permutations of n distinct objs. take n w/ r groups of indistinct objs. = $(n!)/(n_1! \dots n_r!)$

${}^n P_r = n!$ and ${}^n C_r = (n+r-1) C_r$: for perms and

combs where k objs are taken at a time $(a+b)^n = \text{Sum over } k ({}^n C_k a^k b^{(n-k)})$ where $k=0, \dots, n$

Binomial coeff. identity: ${}^n C_k = (n-1) C_{(k-1)} + (n-1) C_k$ where first term maps to A and

second to A^C

$${}^n C_m = {}^n C_{n-m}$$

Sum over r $({}^n C_r (-1)^r (1)^{(n+r)})$ is 0 where $r=0, \dots, n$

Sum over r $(({}^n C_r)^2) = 2^n C_n$ where $r=0, \dots, n$

Sum over s $({}^s C_m) = {}^{(n+1)} C_{-(m+1)}$ where $s=m, \dots, n$

Hypergeometric expansion: $(n+m) C_r =$

$${}^n C_0 {}^m C_r + {}^n C_1 {}^m C_{(r-1)} + \dots + {}^n C_r {}^m C_0$$

a CE and ME enumeration

$n! = (n/e)^n \times \text{root}(2n \times \pi)$ - Stirling's approx. for n!

Trinomial expansion: $(a+b+c)^n = \text{sum over } i, j, k (C^i a^i b^j c^k)$ where $i, j, k=0, \dots, n$ and $i+j+k=n$ and $C^i = n!/(i!j!k!)$

In n-nomial expansion $(a_1 + \dots + a_r)^n$, the # of terms in the sum is ${}^r C_n = (r+n-1) C_n = (r+n-1) C_{(r-1)}$

Expectation, Variance, RV Functions

Expected value of X, ie: $EX/E[X]/\mu_X =$

sum over all $x_k \in R_X (x_k \cdot P(X = x_k))$. It is linear

$$E[aX + b] = aE[X] + b, \ \forall a, b \in \mathbb{R}$$

$$E[X_1 + \dots + X_n] = E[X_1] + \dots + E[X_n]$$

If X is an RV and $Y=g(X)$, then Y is also an RV.



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Expectation, Variance, RV Functions (cont)

$R_Y = \{g(x) \mid x \in R_X\}$ and $P_Y(y) = \text{sum over all } x: g(x)=y (P_X(x))$

$E[g(X)] = \text{sum over all } x_k \in R_X (g(x_k) \cdot P_X(x_k))$

$P_X(x_k)$ (LOTUS)

$\text{Var}(X) = E[(X - \mu_X)^2] = \text{sum over all } x_k \in R_X ((x_k - \mu_X)^2 \cdot P_X(x_k))$

$\text{SD}(X)/\sigma_X = \sqrt{\text{Var}(X)}$

$\text{Covariance} = \text{Cov}(X, Y) = E[XY] - E[X] \cdot E[Y]$,

which will be 0 if X & Y are independent

$\text{Var}(X) = \text{Cov}(X, X) = E[X^2] - (E[X])^2$

$\text{Var}(aX + b) = a^2 \text{Var}(X)$, and if $X = X_1 + \dots +$

X_n , then $\text{Var}(X) = \text{Var}(X_1) + \dots + \text{Var}(X_n)$

$\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) +$

$2ab \text{Cov}(X, Y)$

$\text{Var}(\text{total up to } X_n) = \text{sum of all } \text{Var}(X_i)$ if X_i

is mutually independent for $i = 1 \dots n$.

Summing up over the same conditions for expected values holds true, regardless of independence or not

$\text{Correlation coefficient} = \text{Cov}(X, Y) / (\text{SD}(X) \cdot$

$\text{SD}(Y))$ - ranges between -1 and 1 (inclusive for both limits)

Z-standardized transformation: $Z = (X -$

$\mu_X) / \text{SD}(X)$ - zero mean and unit variance



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