Definitions

Sample Space	The set of all possible outcomes of an experiment is called the sample space and is denoted by Ω.		
Sigma field	A collection of sets F of Ω is called a σ-field if it satisfies the following conditions:		
	1. ∅ ∈ F 2. If A1,...,∈ F then U∞1 Ai ∈ F		3. If A ∈ F then A^c ∈ F
Probability	A probability measure P on (Ω, F) is a function P : F → [0, 1] which satisfies:		
	1.P(Ω)=1 and P(∅)=0	2.	
Conditional Probability	Consider probability space (Ω, F , P) and let A, B ∈ F with P(B) > 0. Then the conditional probability that A occurs given B occurs is defined to be: P(A|B) = P(A ∩ B) / P(B)		
Total Probability	A family of sets B1, . ., Bn is called a partition of Ω if: ∀i !=j Bi ∩Bj =∅ and U∞1 Bi =Ω	P(A) = ∑n1 P(A|Bi)P(Bi)	P(A) = ∑n1 P(A∩Bi)
Independence	Consider probability space (Ω, F , P) and let A, B ∈ F . A and B are independent if P(A ∩ B) = P(A)P(B)		
	More generally, a family of F−sets A1,...,An (∞ > n ≥ 2) are independent if P(∩n1 Ai) = ∏ n1 P(Ai)		

Definitions (cont)			
Random Variable (RV)	A RV is a function X : Ω → R such that for each x ∈ R, {ω ∈ Ω : X(ω) ≤ x} ∈ F. Such a function is said to be F−measurable		
Distribution Function	Distribution function of a random variable X is the function F : R → [0, 1] given by F(x)=P(X ≤x), x∈R.		
Discrete RV	A RV is said to be discrete if it takes values in some countable subset X = {x1,x2,...} of R		
PMF	PMF of a discrete RV X, is the function f :X→[0,1] defined by f(x)=P(X =x). It satisfy:	PDF	function f is called the probability density function (PDF) of the con- tinuous random variable X
	1. set of x s.t. f(x) != 0 is countable		f(x) = F'(x)
	2. ∑ x∈X f(x) = 1		F(x) = ∫-∞x f(u) du
	3. f(x) ≥ 0		
Independence	Discrete RV X and Y are indie if the events {X = x} & {Y =y} are indie for each(x,y)∈X×Y	The RV X and Y are indie if {X≤x} {Y≤y} are indie events for each x, y ∈ R	
	P(X,Y) = P(X=x)P(Y=y)		
	f(x,y) = f(x)f(y)	f(x,y) = f(x)f(y) F(x,y) v	
	E[XY] = E[X]E[Y]		
Expectation	expected value of RV *X* on X,	The expectation of a continuous random variable X with PDF f is given by	
	E[*X*] = ∑x∈X xf(x)	E[*X*] = ∫x∈X xf(x) dx	
	E[g(x)] = ∑x∈X g(x)f(x)	E[g(x)] = ∫x∈X g(x)f(x) dx	
Variance	spread of RV	$E[(X - E[X])^2]$	$E[X^2] - E[X]^2$
MGF (uniquely characterises distribution)	$M(t) = E[e^{Xt}]$ = ∑x∈X e^{Xt} f(x)	t∈T s.t. t for ∑x∈X e^{Xt} f(x) < ∞	
	$M(t) = E[e^{Xt}]$ = ∫x∈X e^{Xt} f(x) dx	t∈T s.t. t for ∫x∈X e^{Xt} f(x) dx < ∞	

Definitions (cont)

	$M(t1,t2) = E[e^{Xt1+Yt2}] = \int z\ e^{Xt1+Yt2} f(x,y)\ dxdy\ (t1,t2)\in T$	$E[X] = \partial/\partial t1\ M(t1,t2)\ \|t1=t2=0$	$E[XY] = \partial^2/\partial t1\partial t2\ M(t1,t2)\ \|t1=t2=0$
	$E[X^k] = M^k(0)$		
Moment	Given a discrete RV X on X, with PMF f and $k \in Z^+$, the k^{th} moment of X is	$E[X^k]$	
Central Moment	k^{th} central moment of X is	$E[(X - E[X])^k]$	
Dependence	Joint distribution function $F : R^2 \to [0,1]$ of X,Y where X and Y are discrete random variables, is given by F(x,y) = P(X≤x∩Y≤y)	The joint distribution function of X and Y is the function F : R2 → [0, 1] given by F(x,y)=P(X≤x,Y ≤y)	
	Joint mass function f : R2 → [0, 1] is given by f(x,y) = P(x∩y)	The random variables are jointly continuous with joint PDF f : R2 → [0, ∞) if F(x, y) = ∫-∞y∫-∞x f(u,v) dudv	
		$f(x,y) = \partial^2/\partial x\partial y\ F(x,y)$	
Marginal	f(x) = ∑y∈Y f(x,y)	f(x) = ∫y∈Y f(x,y)dy	F(x) = lim y->∞ F(x,y) F(x) = ∫-∞x∫-∞∞ f(u,y) dydu
	E[*g(x,y)*] = ∑x,y∈*X*x*Y* g(x,y)f(x,y)	E[*g(x,y)*] = ∫x,y∈*X*x*Y* g(x,y)f(x,y) dxdy	
Covariance	indie => E[XY] = E[X]E[Y], Cov = 0 => ρ = 0	ρ = 0 => E[XY] = E[X]E[Y]	
	Cov[X,Y] = E[(X − E[X])(Y − E[Y])]	Cov[X,Y] = E[XY] - E[X]E[Y]	
Correlation	Gives linear relationship (+/-). |ρ| close to 1 is strong, close to 0 is weak	special for bi-variate normal, indie <=> uncorrelated	
	ρ(X,Y)= Cov[X,Y] / sqrt(Var[X]Var[Y])		
Conditional distribution	The conditional distribution function of Y given X, written FY |x(·|x), is defined by	F(y|x) = ∫-∞y f(x,v)/f(x) dv	f(y|x) = f(x,y)/f(x) where f(x) = ∫-∞∞ f(x,y) dy

Definitions (cont)

	$F_{Y\mid X}(y\mid x) = P(Y \le y \mid X = x)$		
	for any x with $P(X = x) > 0$. The conditional PMF of Y given X =x is defined by ... when x is s.t. $P(X = x) > 0$		
	$f(y\mid x) = P(Y = y \mid X = x)$		
	$f(x,y) = f(x\mid y)f(y)$ or $f(y\mid x)f(x)$		
Conditional expectation	The conditional expectation of a RV Y, given X = x is $E[Y\mid X = x] = \sum_{y\in Y} y f(y\mid x)$ given that the conditional PMF is well-defined	$E[h(X)g(Y)] = E[E[g(Y)\mid X]h(X)] = \int(\int g(Y)f(Y\mid X)\,dx)\,h(X)f(x)\,dx$	
	$E[Y\mid X = x] = \sum_{y\in Y} y f(y\mid x)$	$E[E[Y\mid X]] = E[Y]$	$E[E[Y\mid X]g(X)] = E[Yg(X)]$
	$E[(aX + bY)\mid Z] = aE[X\mid Z] + bE[Y\mid Z]$		
	if X and Y are independent	$E[X\mid Y] = E[X]$	$Var[X\mid Y] = E[X^2\mid Y] - E[X\mid Y]^2$

Theorems

Bayes Theorem	Consider probability space (Ω, F , P) and let A, B ∈ F with P(A), P(B) > 0. Then we have:
	$P(B\mid A) = P(A\mid B)P(B) / P(A)$
Independence	If X and Y are indie RV and g : X → R, h : Y → R, then the RV g(X) and h(Y) are also indie
Expectations	1. if X≥0, E[X]≥0
	2. if a, b∈R then E[aX+bY]=aE[X]+bE[Y]
	3. if X = c∈R always, then E[X]=c.
Variance	1. For a ∈ R, $Var[aX] = a^2 Var[X]$
	2. Uncorrelated Var[X + Y] = Var[X] + Var[Y]
Conditional Expectation	Conditional expectations satisfies $E[E[Y\mid X]] = E[Y]$ assuming all the expectations exist
	for any g : R → R, $E[E[Y\mid X]g(X)] = E[Yg(X)]$ assuming all expectations exist
Change of variable	If (X1,X2) have joint density f(x,y) on Z, then for (Y1,Y2) = T(X1,X2), with T as described above, the joint density of (Y1,Y2), denoted g is: $g(y1,y2)=f(T^{-1}(y1,y2),T^{-1}(y1,y2))\ \mid J(y\ ,y\ )\mid\ (y1,y2)\in T$