

Solving Trigonometric Proofs

Method 1

Pick one side of the equation (usually the most complicated side) and work with that side until it is equal to the other side

Method 2

Work with both sides simultaneously until they are both equal to the same expression.

Helpful Techniques

Rewrite

Rewrite the expression in terms of sine and cosine only

Multiply by One

Multiply the numerator and denominator of a rational expression by a carefully chosen "one"

Combine fractions

Write sums and differences of rational expressions as a single fraction

Factor

Factor trigonometric expressions, using "u-substitution" as needed

Pythagorean Theorem

Look for combinations or portions of Pythagorean Identities. Remember that you can multiply, divide, add or subtract the identity to get a new version.

Goal

Always keep the goal in mind. As you manipulate one side of the equation, keep the other side in mind. Look for commonalities

Verifying Trigonometric Identities

Video: <http://youtu.be/Rf05H8ogHLg>

Pythagorean Identities

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \sin^2 x &= 1 - \cos^2 x \\ \cos^2 x &= 1 - \sin^2 x \\ 1 + \cot^2 x &= \csc^2 x \\ \cot^2 x &= \csc^2 x - 1 \\ \tan^2 x + 1 &= \sec^2 x \\ \tan^2 x &= \sec^2 x - 1 \end{aligned}$$

Divide original Pythagorean identity by $\sin^2 x$ or $\cos^2 x$ to get other identities, subtract to get even more.

Basic Trigonometric Functions

$$\begin{aligned} \sin(-x) &= -\sin(x) & \tan(x) &= \sin(x) / \cos(x) & \csc(x) &= 1/\sin(x) \\ \cos(-x) &= \cos(x) & \cot(x) &= \cos(x) / \sin(x) & \sec(x) &= 1/\cos(x) \\ \tan(-x) &= -\tan(x) & & & \cot(x) &= 1/\tan(x) \end{aligned}$$

Complementary Angle Identities

$$\begin{aligned} \sin(\pi/2 - x) &= \cos(x) \\ \cos(\pi/2 - x) &= \sin(x) \\ \tan(\pi/2 - x) &= \cot(x) \end{aligned}$$

Sum and Difference Identities

$$\begin{aligned} \sin(a \pm b) &= \sin(a) \cos(b) \pm \cos(a) \sin(b) \\ \cos(a \pm b) &= \cos(a) \cos(b) \mp \sin(a) \sin(b) \end{aligned}$$

Use $\sin(a \pm b) / \cos(a \pm b)$ to find $\tan(a \pm b)$

Double Angle Identities

$\sin(2x)$	$\cos(2x)$
$2 \sin(x) \cos(x)$	$\cos^2(x) - \sin^2(x)$
	$1 - 2 \sin^2(x)$
	$2 \cos^2(x) - 1$

