

σ-algebras & Borel sets	
$A \subseteq 2^\Omega$ is a σ-algebra	if $\Omega \in A, A \in A \Rightarrow A^c \in A$ , and $A_i \in A \Rightarrow \cup_{i \in \mathbb{N}} A_i \in A$
$\sigma(E)$	intersection of all σ-algebras containing E
$\mathcal{B}(V)$	$\sigma(\text{open subsets of } V)$

Measures	
Measure	$\mu(\emptyset)=0$ and $\mu(\cup A_i)=\sum \mu(A_i)$ for disjoint $A_i$
Probability	$P(\Omega)=1$
Product	$(\otimes \mu_i)(A_1 \times \dots \times A_n) = \prod \mu_j(A_j)$
Lebesgue	$\lambda^d(\prod_j (a_j, b_j)) = \prod_j (b_j - a_j)$

Measurability	
Measurable	$f^{-1}(A_2) \in \mathcal{A}_1$ for all $A_2 \in \mathcal{A}_2$
Check on generators	$\mathcal{A}_2 = \sigma(\mathcal{E})$ , enough to check $f^{-1}(E) \in \mathcal{A}_1$ for $E \in \mathcal{E}$
Strongly measurable	$f_n$ simple, $f_n \rightarrow f$ pointwise or $\mu$ -a.e.
Pettis	$f$ strongly measurable $\Leftrightarrow f$ separably valued and $\langle f, v' \rangle$ measurable for every $v' \in V$

Bochner integration, change of variables	
$\mu$ -simple	$f = \sum_{j=1}^n 1_{A_j} v_j, \mu(A_j) < \infty$ $\int f d\mu = \sum \mu(A_j) v_j$ for simple $f$
Bochner definition	$f_n \rightarrow f$ $\mu$ -a.e. and $\int \ f - f_n\  d\mu \rightarrow 0$
Bochner criterion	$f$ strongly $\mu$ -measurable, $\int \ f\  d\mu < \infty$

Bochner integration, change of variables (cont)	
Norm bound	$\ \int f d\mu\  \leq \int \ f\  d\mu$
Duality	$\langle \int f d\mu, v' \rangle = \int \langle f, v' \rangle d\mu$
$L^p$	$\mu$ -a.e. equivalence classes of strongly $\mu$ -measurable functions with finite $L^p$ norm
$dv/d\mu$	density of $\nu$ w.r.t $\mu, \nu \ll \mu$
Pushforward	$T_{\#} \mu(A) = \mu(T^{-1}(A))$ $\int f d(T_{\#} \mu) = \int f \circ T d\mu$

Banach-valued RVs	
Random variable	$X: (\Omega, \mathcal{A}) \rightarrow (V, \mathcal{B}(V))$ measurable
$P_X = X_{\#} P$	so $P[X \in B] = P_X(B)$
$\sigma(X)$	$\{X^{-1}(B) : B \in \mathcal{B}(V)\}$
$E[X]$	$= \int_{\Omega} X(\omega) dP(\omega)$ , if $\int \ X\  dP < \infty$
$E[\varphi(X)]$	$= \int_{\Omega} \varphi(X(\omega)) dP(\omega) = \int_V \varphi(v) dP_X(v)$

Conditional probability & independence	
$P[A B]$	$= P[A \cap B] / P[B], P[B] > 0$
A, B independent	$\Leftrightarrow P[A \cap B] = P[A]P[B]$
$X_i$ independent	$\Leftrightarrow \sigma(X_i)$ independent
If $X_1, \dots, X_n$ independent and integrable:	$E[\prod X_i] = \prod E[X_i]$

Conditional expectation	
For simple $Y$	$E[X Y](\omega) = 1/P[A_j] \int_{A_j} X dP$ for $\omega \in A_j$ $= \sum 1_{A_j} Y_j$
$Z = E[X \mathcal{F}]$	iff $Z$ is $\mathcal{F}$ -measurable and $\int_B Z dP = \int_B X dP$ for all $B \in \mathcal{F}$
$E[X Y] := E[X \sigma(Y)], P[A Y] := E[1_A \sigma(Y)]$	

Regular conditional distributions & Doob-Dynkin	
Goal:	define $P[X \in B   Y=y]$ even when $P\{Y=y\}=0$
$\kappa_{X F}: \Omega \times \mathcal{B}(V) \rightarrow [0, 1]$	
For $A \in \mathcal{F}, B \in \mathcal{B}(V): P[A \cap \{X \in B\}] = \int_A \kappa_{X F}(\omega, B) dP(\omega)$	
Doob-Dynkin	$\kappa \sigma(Y)$ -measurable $\Leftrightarrow \kappa = \tau \circ Y$ for measurable $\tau$
$T_{X Y}(y, B) := P[X \in B   Y=y]$	

Conditional densities	
$f_Y(y)$	$= \int_{\mathbb{R}^m} f_{X,Y}(x,y) dx$
$f_{X Y}(x y)$	$= f_{X,Y}(x,y) / f_Y(y)$ , for $f_Y(y) > 0$
$P[X \in B   Y=y]$	$= \int_B f_{X Y}(x y) dx$

Gaussians	
$X \sim N(\mu, \sigma^2)$ :	$f_X(x) = 1/\sqrt{2\pi\sigma^2} \exp(-(x-\mu)^2/(2\sigma^2))$
$X \sim N(\mu, \Sigma)$ :	$f_X(x) \propto \det(\Sigma)^{-1/2} \exp(-1/2 (x-\mu)^T \Sigma^{-1} (x-\mu))$
Gaussian facts	marginals Gaussian; conditionals Gaussian; jointly Gaussian + uncorrelated $\Leftrightarrow$ independent

Distances & Divergences	
$D_{TV}(P, Q)$	$= \sup_{A \in \mathcal{A}}  P(A) - Q(A) $
$D_H(P, Q)$	$= (1/\sqrt{2}) (\int (\sqrt{dP/d\mu} - \sqrt{dQ/d\mu})^2 d\mu)^{1/2}$
$D_{KL}(P  Q)$	$= \int \log(dP/dQ) dP$ if $P \ll Q; \infty$ otherwise
Expectation bound	$\ E_P[f] - E_Q[f]\  \leq 2D_H(P, Q)$ $(E_P\ f\ ^2 + E_Q\ f\ ^2)^{1/2}$



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Not published yet.  
Last updated 6th June, 2026.  
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