Cheatography

Integral Calculus (Various) Cheat Sheet

by Lipsum via cheatography.com/151963/cs/42793/

Derivatives

- Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$
- Product Rule: $(uv)' = u'v + uv'$
- Quotient Rule: $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
- Chain Rule: $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

Trigonometric Identities

- Pythagorean Identities: $\sin^2(x) + \cos^2(x) = 1$
- Sum and Difference Identities: $\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
- Double Angle Identities: $\sin(2x) = 2\sin(x)\cos(x)$

Logarithmic Properties

Logarithmic Properties:

- Logarithmic properties are rules that govern the behavior of logarithmic functions. Some key properties include:
  - Product Rule: $\log_b(xy) = \log_b(x) + \log_b(y)$
  - Quotient Rule: $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
  - Power Rule: $\log_b(x^n) = n\log_b(x)$

Approximating Area Under a Curve

- Left, Right, Midpoint Approximations:
  - Left Sum $= \sum_{i=1}^{n} f(x_{i-1})\Delta x$
  - Right Sum $= \sum_{i=1}^{n} f(x_i)\Delta x$
  - Midpoint Sum $= \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$, where $\bar{x}_i$ is the midpoint of the ith interval.
- Trapezoidal Rule:
  - Trapezoidal Rule $= \frac{\Delta x}{2}[f(a) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(b)]$
- Simpson's Rule:
  - Simpson's Rule $= \frac{\Delta x}{3}[f(a) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(b)]$

Power Sum Rules

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$,
- $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$,
- $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$.

Definite Integrals

- **Definition:** $\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_{i-1})\Delta x$.
- **Properties:** Flip bounds (negate), break into smaller integrals, comparison test.
- **Calc 1 Technique:** Fundamental understanding of integration and its geometric interpretation as area under a curve.

u-substitution

- Replace x with $u(x)$, convert bounds, $dx = \frac{du}{u'(x)}$.
- **Calc 1 Technique:** Use when an integral's integrand is a composite function, resembling the chain rule's application in reverse.

Integration by Parts

- $\int u\,dv = uv - \int v\,du$
- LIATE Rule: Choose u as the algebraic function that appears first in the list of functions: Logarithmic, Inverse trigonometric, Algebraic, Trigonometric, Exponential.

Integrals of Trigonometric Functions

Integrals of Basic Trigonometric Functions:

- $\int \sin(x)\,dx = -\cos(x) + C$
- $\int \cos(x)\,dx = \sin(x) + C$
- $\int \tan(x)\,dx = -\ln|\cos(x)| + C$

Integrals involving Secant and Cosecant:

- $\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$
- $\int \csc(x)\,dx = -\ln|\csc(x) + \cot(x)| + C$

Integrals involving Products of Trigonometric Functions:

- $\int \sin^n(x)\cos^m(x)\,dx$ can often be solved using trigonometric identities or by applying reduction formulas for powers of sine and cosine.

Trigonometric Substitution:

- Used to solve integrals involving radicals or quadratic expressions by substituting trigonometric functions for variables.
- For example, if $\int \sqrt{a^2 - x^2}\,dx$ appears, you could let $x = a\sin(\theta)$ or $x = a\cos(\theta)$ to simplify the integral.

Hyperbolic Trig Functions

- Hyperbolic functions are analogs of circular trigonometric functions, but they are based on hyperbolas instead of circles. The primary hyperbolic functions are:
  - Hyperbolic Sine: $\sinh(x) = \frac{e^x - e^{-x}}{2}$
  - Hyperbolic Cosine: $\cosh(x) = \frac{e^x + e^{-x}}{2}$
  - Hyperbolic Tangent: $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$
- Hyperbolic functions have properties similar to trigonometric functions, such as even/odd symmetry and relationships between their derivatives and themselves.

Power Series

- $\sum_{n=a}^{\infty} a_n(x-a)^n$ with a radius of convergence R.

Order of Convergence

- $\lim_{n\to\infty} \frac{|x_{n+1} - L|}{|x_n - L|^p} = C$, where C is a constant and p is the order of convergence.

Convergence

Convergence and Divergence:

- A series is said to converge if the sequence of partial sums approaches a finite limit as the number of terms increases indefinitely. If the limit exists, it is called the sum of the series.
- Conversely, a series diverges if the sequence of partial sums does not approach a finite limit. In this case, the series does not have a sum.
- Example of convergent series: $\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$, known as the geometric series.
- Example of divergent series: $\sum_{n=1}^{\infty} n$ diverges, as the partial sums increase without bound.

Convergence Tests:

- Convergence tests are used to determine whether a series converges or diverges. Some common convergence tests include:
  - Divergence Test: If $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
  - Geometric Series Test: $\sum_{n=1}^{\infty} ar^n$ converges to $\frac{a}{1-r}$ if $|r| < 1$.
  - Integral Test: If $f(x)$ is positive, continuous, and decreasing for $x \geq 1$ and $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ either both converge or both diverge.

Types of Convergence:

- Absolute Convergence: A series $\sum_{n=1}^{\infty} |a_n|$ converges.
- Conditional Convergence: A series $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} |a_n|$ diverges.

Sequences and Series

Sequences:

- A sequence is an ordered list of numbers written in a specific order. It can be finite or infinite.
- Formally, a sequence is denoted as (a_n) or $\{a_n\}$, where a_n represents the nth term of the sequence.
- Example: $1, 1/2, 1/3, 1/4, \ldots$ is a sequence with the formula $a_n = \frac{1}{n}$.

Series:

- A series is the sum of the terms of a sequence. It represents the total when an infinite number of terms are added together.
- Formally, a series is denoted as $\sum_{n=1}^{\infty} a_n$ or $a_1 + a_2 + a_3 + \ldots$.
- Example: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$ represents the series of the sequence mentioned earlier.

Partial Fractions

- Partial fraction decomposition is a technique used to break down a rational function into simpler fractions.
- The general process involves:
  - 2.1. Factorizing the denominator into linear and irreducible quadratic factors.
  - 2.2. Writing the partial fraction decomposition with undetermined coefficients for each factor.
  - 2.3. Determining the values of the undetermined coefficients by equating coefficients.
- Example: $\frac{3x^2+5x+7}{x^3-2x^2+x}$ can be decomposed into partial fractions as $\frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$.

Logistical Growth Function

- $P(t) = \frac{K}{1+ae^{-rt}}$

r = growth rate, K = carrying capacity

Improper Integrals

- Example: $\int_1^{\infty} \frac{1}{x^p}\,dx$ converges if $p > 1$ and diverges if $p \leq 1$.

Integrals where the limits of integration are infinite or where the function being integrated is undefined at certain points within the interval.

Taylor Polynomials

- A Taylor polynomial of degree n for a function $f(x)$ centered at a point $x = a$ is a polynomial that approximates $f(x)$ near $x = a$.
- The general form of a Taylor polynomial is:
  $P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$
- Here, $f^{(n)}(a)$ denotes the nth derivative of $f(x)$ evaluated at $x = a$, and $n!$ represents the factorial of n.

Taylor's Theorem

- Taylor's Theorem provides a way to quantify the error between a function $f(x)$ and its Taylor polynomial approximation $P_n(x)$.
- It states that if $f(x)$ is $(n+1)$-times differentiable on an interval containing $x = a$, then there exists a number c between a and x such that the error $R_n(x) = f(x) - P_n(x)$ can be expressed as:
  $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$
- This error term is known as the remainder or residual term of the Taylor polynomial.

Maclaurin Series

- A Maclaurin series is a special case of a Taylor series where the polynomial is centered at $x = 0$ (i.e., $a = 0$).
- The Maclaurin series expansion of a function $f(x)$ is given by:
  $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$
- Essentially, the Maclaurin series represents the Taylor series expansion of a function around the point $x = 0$.

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