Cheatography

Integral Calculus (Various) Cheat Sheet

by Lipsum via cheatography.com/151963/cs/42793/

Derivatives

- Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$
- Product Rule: $(uv)' = u'v + uv'$
- Quotient Rule: $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
- Chain Rule: $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

Trigonometric Identities

- Pythagorean Identities: $\sin^2(x) + \cos^2(x) = 1$
- Sum and Difference Identities: $\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
- Double Angle Identities: $\sin(2x) = 2\sin(x)\cos(x)$

Logarithmic Properties

Logarithmic Properties:

- Logarithmic properties are rules that govern the behavior of logarithmic functions. Some key properties include:
  - Product Rule: $\log_b(xy) = \log_b(x) + \log_b(y)$
  - Quotient Rule: $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
  - Power Rule: $\log_b(x^n) = n\log_b(x)$

Approximating Area Under a Curve

- Left, Right, Midpoint Approximations:
  - $\text{Left Sum} = \sum_{i=1}^{n} f(x_{i-1})\Delta x$
  - $\text{Right Sum} = \sum_{i=1}^{n} f(x_i)\Delta x$
  - $\text{Midpoint Sum} = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$, where $\bar{x}_i$ is the midpoint of the ith interval.
- Trapezoidal Rule:
  - $\text{Trapezoidal Rule} = \frac{\Delta x}{2}[f(a) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(b)]$
- Simpson's Rule:
  - $\text{Simpson's Rule} = \frac{\Delta x}{3}[f(a) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(b)]$

Power Sum Rules

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$,
- $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$,
- $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$.

Definite Integrals

- **Definition:** $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_{i-1})\Delta x$.
- **Properties:** Flip bounds (negate), break into smaller integrals, comparison test.
- **Calc 1 Technique:** Fundamental understanding of integration and its geometric interpretation as area under a curve.

u-substitution

- Replace x with $u(x)$, convert bounds, $dx = \frac{du}{u'(x)}$.
- **Calc 1 Technique:** Use when an integral's integrand is a composite function, resembling the chain rule's application in reverse.

Integration by Parts

- $\int u\,dv = uv - \int v\,du$
- LIATE Rule: Choose u as the algebraic function that appears first in the list of functions: Logarithmic, Inverse trigonometric, Algebraic, Trigonometric, Exponential.

Integrals of Trigonometric Functions

Integrals of Basic Trigonometric Functions:

- $\int \sin(x)\,dx = -\cos(x) + C$
- $\int \cos(x)\,dx = \sin(x) + C$
- $\int \tan(x)\,dx = -\ln|\cos(x)| + C$

Integrals involving Secant and Cosecant:

- $\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$
- $\int \csc(x)\,dx = -\ln|\csc(x) + \cot(x)| + C$

Integrals involving Products of Trigonometric Functions:

- $\int \sin^n(x)\cos^m(x)\,dx$ can often be solved using trigonometric identities or by applying reduction formulas for powers of sine and cosine.

Trigonometric Substitution:

- Used to solve integrals involving radicals or quadratic expressions by substituting trigonometric functions for variables.
- For example, if $\int \sqrt{a^2 - x^2}\,dx$ appears, you could let $x = a\sin(\theta)$ or $x = a\cos(\theta)$ to simplify the integral.

Hyperbolic Trig Functions

- Hyperbolic functions are analogs of circular trigonometric functions, but they are based on hyperbolas instead of circles. The primary hyperbolic functions are:
  - Hyperbolic Sine: $\sinh(x) = \frac{e^x - e^{-x}}{2}$
  - Hyperbolic Cosine: $\cosh(x) = \frac{e^x + e^{-x}}{2}$
  - Hyperbolic Tangent: $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$
- Hyperbolic functions have properties similar to trigonometric functions, such as even/odd symmetry and relationships between their derivatives and themselves.

Power Series

- $\sum_{n=0}^{\infty} a_n(x-a)^n$ with a radius of convergence R.

Order of Convergence

- $\lim_{n\to\infty} \frac{|x_{n+1} - L|}{|x_n - L|^p} = C$, where C is a constant and p is the order of convergence.

Convergence

Convergence and Divergence:

- A series is said to converge if the sequence of partial sums approaches a finite limit as the number of terms increases indefinitely. If the limit exists, it is called the sum of the series.
- Conversely, a series diverges if the sequence of partial sums does not approach a finite limit. In this case, the series does not have a sum.
- Example of convergent series: $\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$, known as the geometric series.
- Example of divergent series: $\sum_{n=1}^{\infty} n$ diverges, as the partial sums increase without bound.

Convergence Tests:

- Convergence tests are used to determine whether a series converges or diverges. Some common convergence tests include:
  - Divergence Test: If $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
  - Geometric Series Test: $\sum_{n=1}^{\infty} ar^n$ converges to $\frac{a}{1-r}$ if $|r| < 1$.
  - Integral Test: If $f(x)$ is positive, continuous, and decreasing for $x \geq 1$ and $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ either both converge or both diverge.

Types of Convergence:

- Absolute Convergence: A series $\sum_{n=1}^{\infty} |a_n|$ converges.
- Conditional Convergence: A series $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} |a_n|$ diverges.

Sequences and Series

Sequences:

- A sequence is an ordered list of numbers written in a specific order. It can be finite or infinite.
- Formally, a sequence is denoted as (a_n) or $\{a_n\}$, where a_n represents the nth term of the sequence.
- Example: $1, 1/2, 1/3, 1/4, \ldots$ is a sequence with the formula $a_n = \frac{1}{n}$.

Series:

- A series is the sum of the terms of a sequence. It represents the total when an infinite number of terms are added together.
- Formally, a series is denoted as $\sum_{n=1}^{\infty} a_n$ or $a_1 + a_2 + a_3 + \ldots$.
- Example: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$ represents the series of the sequence mentioned earlier.

Partial Fractions

- Partial fraction decomposition is a technique used to break down a rational function into simpler fractions.
- The general process involves:
  1. Factorizing the denominator into linear and irreducible quadratic factors.
  2. Writing the partial fraction decomposition with undetermined coefficients for each factor.
  3. Determining the values of the undetermined coefficients by equating coefficients.
- Example: $\frac{3x^2+5x-7}{x^3-2x^2+x}$ can be decomposed into partial fractions as $\frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$.

Logistical Growth Function

- $P(t) = \frac{K}{1+ae^{-rt}}$

r = growth rate, K = carrying capacity

Improper Integrals

- Example: $\int_1^{\infty} \frac{1}{x^p}\,dx$ converges if $p > 1$ and diverges if $p \leq 1$.

Integrals where the limits of integration are infinite or where the function being integrated is undefined at certain points within the interval.

Taylor Polynomials

- A Taylor polynomial of degree n for a function $f(x)$ centered at a point $x = a$ is a polynomial that approximates $f(x)$ near $x = a$.
- The general form of a Taylor polynomial is:
  $P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$
- Here, $f^{(n)}(a)$ denotes the nth derivative of $f(x)$ evaluated at $x = a$, and $n!$ represents the factorial of n.

Taylor's Theorem

- Taylor's Theorem provides a way to quantify the error between a function $f(x)$ and its Taylor polynomial approximation $P_n(x)$.
- It states that if $f(x)$ is $(n+1)$-times differentiable on an interval containing $x = a$, then there exists a number c between a and x such that the error $R_n(x) = f(x) - P_n(x)$ can be expressed as:
  $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$
- This error term is known as the remainder or residual term of the Taylor polynomial.

Maclaurin Series

- A Maclaurin series is a special case of a Taylor series where the polynomial is centered at $x = 0$ (i.e., $a = 0$).
- The Maclaurin series expansion of a function $f(x)$ is given by:
  $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$
- Essentially, the Maclaurin series represents the Taylor series expansion of a function around the point $x = 0$.