

### VECTORS

2 vectors equal  $\leftrightarrow$  same magnitude + same direction

norm/magnitude/length of vector  
 $\|v\| = \sqrt{v_1^2 + \dots + v_n^2}$

unit vector  $\rightarrow \|v\| = 1. (v / \|v\|)$

$\|cv\| = |c| \|v\|$

distance  $d(u,v) = \|u-v\|$

Dot product  $u \cdot v = (u_1 v_1 + \dots + u_n v_n)$

$n \cos(\theta) = u \cdot v / (\|u\| \|v\|)$

$u$  &  $v$  are orthogonal when  
 $\text{dot}(u,v) = u \cdot v = 0$

### Equations of Lines/Planes

vector form  $x = x_0 + tv$

parametric in  $R^2$   $x = x_0 + ta, y = y_0 + tb$

parametric in  $R^3$   $x = x_0 + ta, y = y_0 + tb, z = z_0 + tc$

General form of plane  $R^3$   $Ax + By + Cz = D$

point normal eq of a plane  $n \cdot (x - x_0)$

vector eq of a plane  $x = x_0 + sv_1 + tv_2$

### Inverse Matrices

Inversion Algorithm

$[A | I] \rightarrow \dots [I] \dots \rightarrow [I | A^{-1}]$

### Invertible Matrix Theorem

#### Invertible Matrix Theorem

### Invertible Matrix Theorem (cont)

If  $A$  is an  $n \times n$  matrix, and if  $T_A$  is the linear operator on  $R^n$  with standard matrix  $A$ , then the following statements are equivalent.

- $A$  is invertible.
- The RREF of  $A$  is Identity matrix.
- $A$  can be written as a product of elementary matrices.
- $Ax = 0$  has only the trivial solution:  $x = 0$
- $Ax = b$  has exactly one solution for every vector  $b$  in  $R^n$ :  $x = A^{-1}b$ .
- $Ax = b$  is consistent for every vector  $b$  in  $R^n$ .
- The column vectors of  $A$  are linearly independent.
- The row vectors of  $A$  are linearly independent.
- $\det(A) \neq 0$ .
- $\lambda = 0$  is not an eigenvalue of  $A$ .
- $T_A$  is one-to-one.
- $T_A$  is onto.
- The column vectors of  $A$  span  $R^n$ .
- The row vectors of  $A$  span  $R^n$ .
- The column vectors of  $A$  form a basis for  $R^n$ .
- The row vectors of  $A$  form a basis for  $R^n$ .
- $\text{rank}(A) = n$
- $\text{nullity}(A) = 0$
- $A$  has full column rank.

### EigenSTUFF

The scalar  $\lambda$  is called an Eigenvalue of  $A$  when there is a nonzero vector  $x$  such that  $Ax = \lambda x$ .

Vector  $x$  is an Eigenvector of  $A$  corresponding to  $\lambda$ .

The set of all eigenvectors with the zero vector is a subspace of  $R^n$  called the eigenspace of  $\lambda$ .

- Find Eigenvalues:  $\det(\lambda I - A) = 0$
- Find Eigenvectors:  $(\lambda I - A)x = 0$

If  $A$  is a triangular matrix then its eigenvalues are on its main diagonal

