MVE137 - Chalmers University Cheat Sheet

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Basic Probability Definitions

Sample Space (Ω)	Set of all possible outcomes of a random experiment.
Event	Outcome of a random experiment (inside Ω)
σ-field	The allowable events constitute a family of sets F, usually referred to as σ-field. Each set in F is a subset of the sample space Ω.
Probability measure (P)	A probability measure on (Ω, F) is a function P : F → [0, 1] that satisfies the following two properties: 1. P[Ω] = 1 2. The probability of the union of a collection of disjoint members is the sum of its probabilities
Probability space	(Ω, F, P)
Basic properties of probability measures	P[∅] = 0 P[A¯] = 1 − P[A] If A ⊂ B, then P[B] = P[A] + P[B \ A] ≥ P[A] P[A ∪ B] = P[A] + P[B] − P[A ∩ B]
Inclusion Exclusion Principle (comes from last basic property of probability measures)	Given sets A1, A2... **P[union(Ai)] ≤ sum(P[Ai])** When the two events are disjoint, the inequality is = as they don't share any common space: P[A ∩ B] = 0

Basic Probability Definitions (cont)

Sampling strategy	Choose repeatedly a random number in Ω
Sampling with replacement	Select random numbers in Ω, **without taking into account** which ones you've already tested. Therefore, there will be some numbers tested multiple times
Sampling without replacement	Select random numbers in Ω **taking into account** which ones you've already used. Therefore, you won't run the algorithm with the same number more than once
Independent (events or family)	Two events are independent if: P[A ∩ B] = P[A] P[B] It also applies to families {Ai, i∈ I}
Pairwise	To form all possible pairs (two items at a time) from a set
Pairwise independent (family or events)	A family or events are pairwise independent if: **P[Ai ∩ Aj] = P[Ai] P[Aj] for all i != j** In english terms, a family or events is pairwise independent if any of its possible pairs is independent of each other. For example: P(A∩B)=P(A)P(B) P(A∩C)=P(A)P(C) P(B∩C)=P(B)P(C)

Basic Probability Definitions (cont)

Mutually independent (events)	More than two events (i.e. A,B,C) are mutually independent if: 1. They are pairwise independent 2. They meet the condition: P(A ∩ B ∩ C) = P(A) × P(B) × P(C) In plain english, events are mutually independent if any event is independent to the other events
Conditional Probability	If P[B] > 0, the conditional probability that A occurs give that B occurs is: P[A|B]=P[A∩B]/P[B]
Conditional Probability (independent events)	If A and B are independent events, then: P[A|B] = P[A∩ B]/P[B] = (P[A]*P[B])/P[B] = P[A]
Law of Total Probability	Let e1...en be **partitions** of Ω (a collection of ALL the sets in Ω which are independent of each other). Also assuming P[ei] > 0 for all i. The probability of A can be written as: P[A] = sum(i=1,n)(P[A|ei]*P[ei]) In english, it's the sum of all the possible scenarios in which A can occur

Basic Probability Definitions (cont)

Bayes Theorem	Assuming e1...en be **partitions** of Ω: P[ej|B] = P[Ej ∩ B]/P[B] = (P[B|Ej]*P[Ej])/(sum(i=1,n)(P[B|ei]*P[ei]) It's basically using conditional theory and then applying conditional theory again for the top part and law of total probability in the lower part

Discrete Random Variables and Expectation

Random Variable	A random variable X on a sample space Ω is a real-valued (measurable) function on Ω; that is X : Ω → R. Denoted as upper case in this course and real numbers as lower case
Discrete Random Variable	A discrete random variable is a random variable that outputs only a finite or countably infinite number of values (i.e. number of kids in a family, range between 1 and x)
Probability that X=a	Sum of all the events *w* in Ω which X(w) = x

Discrete Random Variables and Expectation (cont)

Independence of random variables	Two random variables X and Y are independent if and only if: P[(X = x)∩(Y = y)] = P[X=x]*P[Y=y] for all values x and y
Mutually independent random variables	Like mutually independent events
Expectation (mean)	It is a weighted average of the values assumed by the random variable, taking into account the probability of getting that value. The expectation of a discrete random variable X, denoted by E[X] is given by E[X] = sum(i=x,X)(x*P[X = x])