

### Basic Probability Definitions

Sample Space ( $\Omega$ )	Set of all possible outcomes of a random experiment.
Event	Outcome of a random experiment (inside $\Omega$ )
$\sigma$ -field	The allowable events constitute a family of sets $F$ , usually referred to as $\sigma$ -field. Each set in $F$ is a subset of the sample space $\Omega$ .
Probability measure (P)	A probability measure on $(\Omega, F)$ is a function $P : F \rightarrow [0, 1]$ that satisfies the following two properties: 1. $P[\Omega] = 1$ 2. The probability of the union of a collection of disjoint members is the sum of its probabilities
Probability space	$(\Omega, F, P)$
Basic properties of probability measures	$P[\emptyset] = 0$ $P[A^c] = 1 - P[A]$ If $A \subset B$ , then $P[B] = P[A] + P[B \setminus A] \geq P[A]$ $P[A \cup B] = P[A] + P[B] - P[A \cap B]$
Inclusion Exclusion Principle (comes from last basic property of probability measures)	Given sets $A_1, A_2, \dots$ <b><math>P[\text{union}(A_i)] \leq \text{sum}(P[A_i])</math></b> When the two events are disjoint, the inequality is = as they don't share any common space: $P[A \cap B] = 0$

### Basic Probability Definitions (cont)

Sampling strategy	Choose repeatedly a random number in $\Omega$
Sampling with replacement	Select random numbers in $\Omega$ , <b>without taking into account</b> which ones you've already tested. Therefore, there will be some numbers tested multiple times
Sampling without replacement	Select random numbers in $\Omega$ <b>taking into account</b> which ones you've already used. Therefore, you won't run the algorithm with the same number more than once
Independent (events or family)	Two events are independent if: $P[A \cap B] = P[A] P[B]$ It also applies to families $\{A_i, i \in I\}$
Pairwise	To form all possible pairs (two items at a time) from a set
Pairwise independent (family or events)	A family or events are pairwise independent if: <b><math>P[A_i \cap A_j] = P[A_i] P[A_j]</math> for all <math>i \neq j</math></b> In english terms, a family or events is pairwise independent if any of its possible pairs is independent of each other. For example: $P(A \cap B) = P(A)P(B)$ $P(A \cap C) = P(A)P(C)$ $P(B \cap C) = P(B)P(C)$

### Basic Probability Definitions (cont)

Mutually independent (events)	More than two events (i.e. $A, B, C$ ) are mutually independent if: 1. They are pairwise independent 2. They meet the condition: $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$ In plain english, events are mutually independent if any event is independent to the other events
Conditional Probability	If $P[B] > 0$ , the conditional probability that A occurs given that B occurs is: $P[A B] = P[A \cap B] / P[B]$
Conditional Probability (independent events)	If A and B are independent events, then: $P[A B] = P[A \cap B] / P[B] = (P[A] \times P[B]) / P[B] = P[A]$
Law of Total Probability	Let $e_1, \dots, e_n$ be <b>partitions</b> of $\Omega$ (a collection of ALL the sets in $\Omega$ which are independent of each other). Also assuming $P[e_i] > 0$ for all $i$ . The probability of A can be written as: $P[A] = \text{sum}(i=1, n)(P[A e_i] \times P[e_i])$ In english, it's the sum of all the possible scenarios in which A can occur



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### Basic Probability Definitions (cont)

Bayes Theorem Assuming  $e_1, \dots, e_n$  be **partitions** of  $\Omega$ :

$$P[e_j|B] = \frac{P[e_j \cap B]}{P[B]} = \frac{P[B|e_j]P[e_j]}{\sum_{i=1}^n (P[B|e_i]P[e_i])}$$

It's basically using conditional theory and then applying conditional theory again for the top part and law of total probability in the lower part

### Discrete Random Variables and Expectation

**Random Variable** A random variable  $X$  on a sample space  $\Omega$  is a real-valued (measurable) function on  $\Omega$ ; that is  $X : \Omega \rightarrow \mathbb{R}$ .  
Denoted as upper case in this course and real numbers as lower case

**Discrete Random Variable** A discrete random variable is a random variable that outputs only a finite or countably infinite number of values  
(i.e. number of kids in a family, range between 1 and  $x$ )

**Probability that  $X=a$**  Sum of all the events  $w$  in  $\Omega$  which  $X(w) = a$

### Discrete Random Variables and Expectation (cont)

**Independence of random variables** Two random variables  $X$  and  $Y$  are independent if and only if:  
 $P[(X = x) \cap (Y = y)] = P[X=x] \cdot P[Y=y]$   
for all values  $x$  and  $y$

**Mutually independent random variables** Like mutually independent events

**Expectation (mean)** It is a weighted average of the values assumed by the random variable, taking into account the probability of getting that value.  
The expectation of a discrete random variable  $X$ , denoted by  $E[X]$  is given by  
 $E[X] = \sum_{i=1}^n (x_i \cdot P[X = x_i])$



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