Solving Problems Using Linear Equations

Worked Example 6:

Mira is twice Sanjin's age. How old are Mira and Sanjin now if in 10 years the sum of their ages will be 89. Use *s* to represent Sanjin's age.

Sanjin's Age: s Mira's present age: 2s

In 10 years:

Sanjin = s + 10 Mira's age = 2s + 10

(s+10) + (2s+10) = 89

3s + 20 = 89

3s = 69

s = 23

2s = 2 x 23 = 46

Sanjin is 23 years old and Mira is 46 years old. In 10 years they will be 33 and 56 old respectively; 33 + 56= 89

Distance and Midpoint

Distance Formula	Midpoint Formula
The distance, *d*, between any two Cartesian coordinates (x1, y1) and (x2, y2) is:	The midpoint M(x,y) between 2 points (x1, y1) and (x2, y2) is given by:
d =	M(x,y) =

Gradient

The gradient of a line can be found by the ratio of the vertical **rise** to the horizontal **run** between any two points on the line.

Gradient AB = (vertical rise)/(horizontal run)

The letter *m* is used to represent the gradient. **y=mx+b**

m = (rise)/(run)

= (change in *y*-value)/(change in *x*-value)

= (y2-y1)/(x2-x1)

Linear Graphs: Gradient and Y-Intercept

The **gradient-intercept form:**

y=mx+b, where *m* is the gradient and *b* is the y-intercept.

The **general form:**

ax+by+c=0, where *a*, *b* and *c* are constants and *a* ≠ 0, b ≠ 0.

Linear Graphs: Gradient and Y-Intercept (cont)

In this equation, the gradient *m* = (-a)/(b) and the y-intercept = (-c)/(b)can be found by rearranging the equation to make *y* the subject.

Worked Example 15

(a) y = -3x+2

gradient = -3

y-intercept: (0,2)

(b) y = x-1

= 1x+(-1)

gradient = 1

y-intercept: (0,-1)

(c) 2y = 5x - 4

y = (5)/(2)x - 2

gradient = (5)/(2)

y-intercept: (0,-2)

(d) y+2x =6

y = 6-2x

y = -2x+6

gradient: -2

y-intercept: (0,6)

Sketching Linear Graphs

Graphing using the y-intercept and gradient

There is no need to plot a number of points to graph linear relationships. Because only 2 points are needed to define a line, use the y-intercept as one point on the graph and the gradient, *m* to locate the second point. Once 2 points are known, a straight line can be drawn through them.

Sketching other Linear Graphs, y=mx+b, b ≠ 0

If the y-intercept is not the origin, then first identify the y-intercept from the equation before using the gradient to find the second point.

Using the equation *y=mx+b* and substituting x = 0, you get y = b.

x = 0 is the x=coordinate of a point on the y-axis, so *b* is the y-value at the point at the point at which the graph crosses the y-axis. This is the y-intercept

Worked Example 17

Sketching Linear Graphs (cont)

Use the y-intercept and the gradient to sketch the graph of y = 2x+1

y-intercept: (0,1)

Gradient = 2

= (2)/(1) = (rise)/(run)

Parallel and Perpendicular Lines

Worked Example 19

Find the gradient of the line parallel to each of the following

(a) y = 2x-5

m = 2

Any parallel line will have a gradient of 2.

(b) 2x + 3y = 6

2x-2x+3y = 6 - 2x

(3y)/(3) = - (2)/(3)x + (6)/(3)

y = - (2)/(3)x + 2

m1 = - (2)/(3)

Any parallel line will have a gradient of - (2)/(3).

Worked Example 20

Find the gradient of a line perpendicular to each of the following

(a) y = 4x + 3

m1 = 4

m2 = - (1)/(m1)

= - (1)/(4)

(b) 3x-5y = 10

3x-3x-5y = 10-3x

-5y = -3x + 10

(-5y)/(-5) = (-3x)/(-5) + (10)/(-5)

y = (3)/(5)x - 2 m1= (3)/(5)

m2 = - (1)/(m1) = -(5)/(3)