UWI Comp2211 - Analysis of Algorithms Cheat Sheet

by Keto via cheatography.com/183106/cs/38117/

Summations - Closed Forms

(1) $\sum_{k=m}^{n} c = (n - m + 1)c.$

(2) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$

(3) $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$

(4) $\sum_{k=0}^{n} a^k = \frac{a^{n+1} - 1}{a - 1}$ (where $a \neq 1$)

(5) $\sum_{k=1}^{n} ka^k = \frac{a - (n+1)a^{n+1} + na^{n+2}}{(a-1)^2}$ (where $a \neq 1$).

Summations - Rules

(1) $\sum ca_k = c \sum a_k.$

(2) $\sum (a_k + b_k) = \sum a_k + \sum b_k$

(3) $\sum a_k x^{i+k} = x^i \sum a_k x^k.$

(4) $\sum_{k=m}^{n} a_{k+i} = \sum_{k=m+i}^{n+i} a_k.$

(5) Collapsing Sums) $\sum_{k=1}^{n} (a_k - a_{k-1}) = a_n - a_0$ and $\sum_{k=1}^{n} (a_{k-1} - a_k) = a_0 - a_n.$

Logarithm Rules

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b(x^a) = a \cdot \log_b(x)$$

$$\log_k(x) = \frac{\ln(x)}{\ln(k)} = \frac{\log_{10}(x)}{\log_{10}(k)}$$

$$a^{\log_b k} = k^{\log_b a}$$

Note: for AoA, $lg = \log_2$

$$e.g.\ lg\, 3 = \frac{\log_{10}(3)}{\log_{10}(2)}$$

Asymptotic Analysis - Common Orders of Growth

Slowest Growth - Fastest Growth

$\Theta(1)$: constant

$\Theta(\log n)$: logarithmic

$\Theta(n)$: linear

$\Theta(n \log n)$:

$\Theta(n^2)$: quadratic

$\Theta(n^k)$ (for constant k) : polynomial

$\Theta(k^n)$ (for constant k) : exponential

Master Theorem Shortcut

Case	Condition	Result
1	k < E	n^E
2	k == E	n^K lg(n)
3	k > E	n^K

Fermat's Little Theorem

For any prime p, for any x:

$x^p \equiv x \pmod{p}$

Alternatively, for any x ≠ 0:

$x^{p-1} \equiv 1 \pmod{p}$

Mod Operations

(x + y) mod n = ((x mod n) + (y mod n)) mod n

(xy) mod n = ((x mod n) × (y mod n)) mod n

(x − y) mod n = ((x mod n) − (y mod n)) mod n