expected variable

The *expected value*, also called the *expectation* or *mean*, of the random variable X on the sample space S is equal to

$$E(X) = \sum_{s \in S} p(s)X(s).$$

The *deviation* of X at $s \in S$ is $X(s) - E(X)$, the difference between the value of X and the mean of X.

expected value of a die: (1/6)*1 + (1/6)*2 + (1/6)*3 + ... (1/6)*6 = 7/2

expected value with large outcomes

If X is a random variable and $p(X = r)$ is the probability that $X = r$, so that $p(X = r) = \sum_{s \in S, X(s)=r} p(s)$, then

$$E(X) = \sum_{r \in X(S)} p(X = r)r.$$

The expected number of successes when n mutually independent Bernoulli trials are performed, where p is the probability of success on each trial, is np.

Geometric Distribution

A random variable X has a *geometric distribution with parameter* p if $p(X = k) = (1-p)^{k-1}p$ for $k = 1, 2, 3, \ldots$, where p is a real number with $0 \le p \le 1$.

If random variable X has the geometric distribution with parameter p, then E(x) = 1/p. (expected value)

Expected value IRV

If X and Y are independent random variables on a sample space S, then E(XY) = E(X)E(Y).

Theorem

Variance

If X is a random variable on a sample space S and E(X) = mu, then V(X) = E((X - mu)^2) V(X) = E(X^2) - E(X)^2.

Variance => how widely the values of expected value of a random value is distributed.

Variance of the number of successes in n Bernoulli trials is *npq*, where *q* is 1 - *p*

Chebyshev's Inequality

CHEBYSHEV'S INEQUALITY Let X be a random variable on a sample space S with probability function p. If r is a positive real number, then

$$p(|X(s) - E(X)| \ge r) \le V(X)/r^2.$$

The likelihood that a random variable takes a value far from it's expected value. This inequality provides an upper bound on the probability that the value of a random variable differs from the expected value of the random variable by more than a specified amount.

Equivalence Relation

A relation on a set A is called an *equivalence relation* if it is reflexive, symmetric, and transitive.

If relations a and b are related by an equivalence relation, they are equivalent, denoted by *a ~ b*

Graph Terminology

TABLE 1 **Graph Terminology.**

Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

.

Bipartite Rules for Special Simple Graphs

(a) For which values of n are these graphs bipartite?
i) K_n ii) C_n W_n Q_n

Answers:

(i)) By the definition given in the text, K_1 does not have enough vertices to be bipartite (the sets in a partition have to be nonempty). Clearly K_2 is bipartite. There is a triangle in K_n for $n > 2$, so those complete graphs are not bipartite. (See Exercise 23.)

(ii)) First we need $n \ge 3$ for C_n to be defined. If n is even, then C_n is bipartite, since we can take one part to be every other vertex. If n is odd, then C_n is not bipartite.

(iii)) Every wheel contains triangles, so no W_n is bipartite.

(iv)) Q_n is bipartite for all $n \ge 1$, since we can divide the vertices into these two classes: those bit strings with an odd number of 1s, and those bit strings with an even number of 1s.

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Strong vs Weak conn.

A directed graph is *strongly connected* if there is a path from *a* to *b* and from *b* to *a* whenever *a* and *b* are vertices in the graph. A directed graph is *weakly connected* if there is a path between every two vertices in the underlying undirected graph.

Strongly connected graph is also weakly connected. Look for strong components (vertices and cycles)

Euler Circuit Rules for Spec. Graphs

For which values of n do these graphs have Euler Circuits?
i) K_n ii) C_n W_n Q_n

Answer:

(i)) The degrees of the vertices $(n-1)$ are even if and only if n is odd. Therefore there is an Euler circuit if and only if n is odd (and greater than 1, of course).

(ii)) For all $n \ge 3$, clearly C_n has an Euler circuit, namely itself.

(iii)) Since the degrees of the vertices around the rim are all odd, no wheel has an Euler circuit.

(iv)) The degrees of the vertices are all n. Therefore there is an Euler circuit if and only if n is even (and greater than 0, of course).

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Euler circuit

An *Euler circuit* in a graph G is a simple circuit containing every edge of G. An *Euler path* in G is a simple path containing every edge of G.

A connected multigraph with at least two vertices has an Euler circuit if and only if each of its vertices has even degree. It has an Euler path but not a circuit if and only if it has exactly two vertices of odd degree.