

table 1

TABLE 7 Logical Equivalences Involving Conditional Statements.

$$\begin{aligned}
 p \rightarrow q &\equiv \neg p \vee q \\
 p \rightarrow q &\equiv \neg q \rightarrow \neg p \\
 p \vee q &\equiv \neg p \rightarrow q \\
 p \wedge q &\equiv \neg(p \rightarrow \neg q) \\
 \neg(p \rightarrow q) &\equiv p \wedge \neg q \\
 (p \rightarrow q) \wedge (p \rightarrow r) &\equiv p \rightarrow (q \wedge r) \\
 (p \rightarrow r) \wedge (q \rightarrow r) &\equiv (p \vee q) \rightarrow r \\
 (p \rightarrow q) \vee (p \rightarrow r) &\equiv p \rightarrow (q \vee r) \\
 (p \rightarrow r) \vee (q \rightarrow r) &\equiv (p \wedge q) \rightarrow r
 \end{aligned}$$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$$\begin{aligned}
 p \leftrightarrow q &\equiv (p \rightarrow q) \wedge (q \rightarrow p) \\
 p \leftrightarrow q &\equiv \neg p \leftrightarrow \neg q \\
 p \leftrightarrow q &\equiv (p \wedge q) \vee (\neg p \wedge \neg q) \\
 \neg(p \leftrightarrow q) &\equiv p \leftrightarrow \neg q
 \end{aligned}$$

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## Proof Laws

TABLE 6 Logical Equivalences.

Equivalence	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation laws

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## Inference

TABLE 1 Rules of Inference.

Rule of Inference	Tautology	Name
$\frac{p}{p \rightarrow q}$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens
$\frac{\neg q}{p \rightarrow q}$ $\therefore \neg p$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$\frac{p \rightarrow q}{p \rightarrow r}$ $q \rightarrow r$ $\therefore p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$\frac{p \vee q}{\neg p}$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
$\frac{p}{p \vee q}$ $\therefore p \vee q$	$p \rightarrow (p \vee q)$	Addition
$\frac{p \wedge q}{p}$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
$\frac{p}{p \wedge q}$ $q$ $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$\frac{p \vee q}{\neg p \vee r}$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

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## Set ID's

TABLE 1 Set Identities.

Identity	Name
$A \cap U = A$ $A \cup \emptyset = A$	Identity laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{\overline{A}} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive laws
$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement laws

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## DeMorgans Quant

TABLE 2 De Morgan's Laws for Quantifiers.

Negation	Equivalent Statement	When Is Negation True?	When False?
$\neg \exists x P(x)$	$\forall x \neg P(x)$	For every $x$ , $P(x)$ is false.	There is an $x$ for which $P(x)$ is true.
$\neg \forall x P(x)$	$\exists x \neg P(x)$	There is an $x$ for which $P(x)$ is false.	$P(x)$ is true for every $x$ .

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## Quant Inference

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	Name
$\frac{\forall x P(x)}{\therefore P(c)}$	Universal instantiation
$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$	Universal generalization
$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$	Existential instantiation
$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$	Existential generalization

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## 2 Var Quant

TABLE 1 Quantifications of Two Variables.

Statement	When True?	When False?
$\forall x \forall y P(x, y)$ $\forall y \forall x P(x, y)$	$P(x, y)$ is true for every pair $x, y$ .	There is a pair $x, y$ for which $P(x, y)$ is false.
$\forall x \exists y P(x, y)$	For every $x$ there is a $y$ for which $P(x, y)$ is true.	There is an $x$ such that $P(x, y)$ is false for every $y$ .
$\exists x \forall y P(x, y)$	There is an $x$ for which $P(x, y)$ is true for every $y$ .	For every $x$ there is a $y$ for which $P(x, y)$ is false.
$\exists x \exists y P(x, y)$ $\exists y \exists x P(x, y)$	There is a pair $x, y$ for which $P(x, y)$ is true.	$P(x, y)$ is false for every pair $x, y$ .

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## Union/Intersect Collection

The union of a collection of sets is the set that contains those elements that are members of at least one set in the collection.

We use the notation

$$A_1 \cup A_2 \cup \dots \cup A_n = \bigcup_{i=1}^n A_i$$

to denote the union of the sets  $A_1, A_2, \dots, A_n$ .

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