sum(term, a next, b)

term: fn applied to each term

term(a): first term

next: applied b - a times

b: no. of terms (0 is returned for the $(b+1)^{th}$ term

sum implementation

def sum(term, a, next, b):

sum(a, b) = t(a) + t(n(a)) + t(n^2(a)) + ... + t($n^{(b-a)}$(a)) + 0

if a > b: return 0

else: return term(a) + sum(term, next(a), next, b)

fold(op, f, n)

n: op applied for **n times**

fold_right:

(f(n) ⨁ (f(n -1) ⨁ (f(n-2) … ⨁ (f(1) ⨁ f(0))

(4 - (3 - (2 - (1 - 0))))

fold_left:

(((f(0) ⨁ f(1)) ⨁ f(2)) ⨁ … f(n) → n + 1 times

Matters for non-associative operations e.g. ((1 - 2) - 3) - 4)

fold_right

fold(..) = op(f(n), fold(n-1))

= f(n) ⊕ fold(n-1)

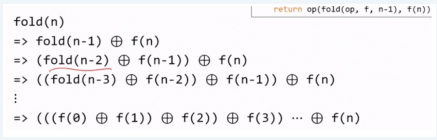
= f(n) ⊕ [f(n-1) ⊕ fold(n-2)]

= f(n) ⊕ f(n-1) ⊕ fold(n-2) --- assumes op is assoc./commu.

= f(n) ⊕ f(n-1) ⊕ f(n-2) ⊕ ... ⊕ f(n-(n-1)) ⊕ fold(n-n)

= f(n) ⊕ f(n-1) ⊕ f(n-2) ⊕ ... ⊕ f(n-(n-1)) ⊕ f(0)

fold_left



accumulate(fn, initial, seq)

>>> accumulate(lambda x, y: x + y, 0, (1, 2, 3, 4, 5))

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>>> accumulate(lambda x, y: (x, y), (), (1, 2, 3, 4, 5))

(1, (2, (3, (4, (5, ())))))

seq contains n elements

when seq[n] == () i.e. index out of range,

initial is returned

enumerate_interval(low, high)

enumerate_interval(2, 7)

> (2, 3, 4, 5, 6, 7)

enumerate_interval(1, 1)

> (1,)

enumerate_interval(4, -1)

> ()

sum and product using fold

def sum_of_int(a, b):

return fold(op = lambda x, y: x + y, f = lambda x: a + x, n = b - a)

a + (a + 1) + (a + 2) + ... + b - 1, b

b = a + (b - a)

def product_int(a, b):

return fold(op = lambda x, y: x * y, f = lambda x: a + x, n = b - a)

fold2(op, term, a, next, b, base)

fold2() = term(a) ⨁ [term(next(a)) ⨁ [.. ⨁ base]

#op applied for (b-a) times

#base occurs when b > a

iterative fold (left)

```
def iter_fold(op, f, n):
        res = f(0)
        for i in range(n):
                res = op(res, f(i + 1))
         return res
```