

Given CDF with two cases, generate X

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F(x) = ( 1/(2)*e^(-x/2) if x < 0; 1 - (1/(2))*e^(-x/2) if x > 0 )

For X < 0 (U < 0.5):
Set: F(X) = (1/(2))*e^(-x/2) = 0
Solve: X = -(1/(2))*ln(2U)

For X > 0 (U > 0.5):
Set: F(X) = 1 - (1/(2))*e^(-x/2) = 0
Solve: X = -(1/(2))*ln(2(1-U))

Final Answer:
X = -(1/(2))*ln(2U) if U < 0.5; 1 - (1/(2))*ln(2(1-U)) if U > 0.5
    
```

Arena Templates

Basic Process		Advanced Process	
Module	Sheet	Module	Sheet
Create	Attribute	Seize	Adv. Set
Dispose	Entity	Delay	Expression
Process	Queue	Release	Failure
Batch	Resource		
Separate	Variable		
Assign	Schedule		
Record	Set		

Blocks		Advanced Transfer	
Module	Sheet	Module	Sheet
Seize		Station	Sequence
Delay		Route	Conveyor
Release		Enter	Transporter
Queue		Leave	Distance Segment

Transporter uses Request/Free, requires Distance Set.
Conveyor uses Access/Exit, requires Segment Set.

Arena Variables and Function

DISC(0.3, 1, 0.8, 2, 1.0, 3)	DISC generates random discrete values based on cumulative probabilities; pair each probability with its corresponding value.
TNOW	Current simulated time
NR(Res)	Res Servers currently in service
NQ(Queue)	Number of customers in Queue
Mod.Nu- mberOut	Customers who have left the mode

Arena Set Types

Resource, Counter, entity type, entity picture

Arena Key Modules

Assign	Give new value to an attribute
Decide	Route customers probabilistically or conditionally
Separate / Clone	Split one customer into two or more clones
Route	Move entities station to station (advanced transfer)
Enter / Leave	Usually paired together for station management
Seize - Delay - Release	equals single Process module
Queue block	Cannot connect with a process module

Example of finding X for Pois

x	f(x) = e ^{-λ} λ ^x /x!	F(x)	Unif(0,1)'s
0	0.8187	0.8187	[0, 0.8187]
1	0.1637	0.9824	(0.8187, 0.9824]
2	0.0164	0.9988	(0.9824, 0.9988]
3	0.0011	0.9999	(0.9988, 0.9999]
≥ 4	0.0001	1.0000	(0.9999, 1.0000]

Notice that I'm not very concerned with the "≥ 4" case. Anyway, suppose that I draw a PRN U = 0.726. What value of X do I get?

- (a) 0
(b) 0.726
(c) 1
(d) 2
(e) 3

Solution: From the table, we see that U = 0.726 clearly corresponds to X = 0, choice (a). □

Universal truths

-ln(U)	~Exponential(1)
lnu	

messy cdf

See F(X)	Replace with Uniform(0,1)
Multiply by 3	U(0,3)
Subtract 1	U(-1,2)
Mean	(-1+2)/2 = 1/2
What is the mean of the random variable 3F(X)-1?	

Inverse Transformation

Given U, find Z	invNorm(U, 0, 1)
Given Z, find U	normCdf(-9999, Z, 0, 1)
Exponential(λ)	X = -ln(1-U)/λ
Uniform(a,b)	X = a + (b-a)·U
Weibull(a,b)	X = a * (-ln(U)) ^{1/b}
Triangular	If U<0.5: √(2U); If U≥0.5: 2-√(2(1-U))
Bernoulli(p)	If U < 1-p → 0; Else → 1
Poisson(λ)	Build CDF, match U
Discrete	[n·U]
Unif(1,n)	
Erlang(k,λ)	-(1/λ)ln(∏Ui)
Geometric	ln(1-U)/ln(1-p)

For discrete: Find smallest x where F(x) ≥ U
For continuous: Use inverse CDF formulas
Box-Muller generates TWO Normal(0,1) values from TWO Uniform(0,1) values

XOR

XOR is only true if different

Expected Value, Variance, ...

Discrete E[X]	SUM[x * f(x)]
Continuous E[X]	SUM[x * f(x) dx]
Variance of X	$E[X^2] - (E[X])^2$
Standard Deviation of X	SQRT[Var(X)]

Expected Value Joint pdf

Joint p.d.f.: $f(x,y) = 2xy^2$; Domain: $0 < x < 1$, $0 < y < 1$

Find: $E[2X-1]$

$$E[2X - 1] = \int_0^1 \int_0^1 (2x - 1) \cdot 2xy^2 dy dx$$

Random Number Generators

Bad generators Midsquare number generator, Random number tables, von Neumann's mid-square method, Fibonacci generator, Additive congruential generator, RANDU

Good generators Linear Congruential Generators (modern cycle length $> 2^{191}$; Mersenne Twister (2^{19937}))

Randu $65539Xi \bmod(2^{31})$

Desert island $16807Xi \bmod(2^{31}-1) \bmod(21-47483647)$

Desert island technique $Z = [\text{SUM}(Ui) - n/2] / [\text{SQRT}(n * 1/12)]$

Inverse Transform Method Key Problems

If $X \sim \text{Normal}(0,1)$, what's the distribution of $\Phi(X)$?
If $U \sim \text{Uniform}(0,1)$ and $\Phi(x)$ is the CDF of the standard normal, what is the distribution of $2\Phi^{-1}(U)+3$?
 $-3\ln(U^2V^2) = -6\ln(U) - 6\ln(V) \sim \text{Exp}(1/6) + \text{Exp}(1/6) \sim \text{Erlang}(2, 1/6)$

Joint Probability Mass Function

$$E[XY] = \text{Summe von } x * y * f(xy)$$

Joint p.m.f.

Are independent if X and Y are independent if and only if $p_{X,Y}(x,y) = p_X(x) \cdot p_Y(y)$

For example $P(X=1, Y=0) = P(Y=0) * P(X=1)$

$$E[XY] = \text{Example: } (1)(0)(0.2) + (1)(1)-(0.0.0) + \dots$$

$$\text{Cov}(X,Y) = E[XY] - E[Y]E[X]$$

$$\text{Variance } X + Y = \text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X,Y)$$

Joint p.m.f. (cont)

$$\text{Variance } X - Y = \text{Var}(X) + \text{Var}(Y) - 2\text{Cov}(X,Y)$$

Theorem $\text{Cov}(X,Y) = 0$ if X, Y independent. Converse not true.

$$\text{Correlation} = \frac{p = \text{Cov}(X,Y) / \text{SQRT}(\text{Var}(X) * \text{Var}(Y))}{}$$

pdf, cdf

pdf $\rightarrow$ cdf integrate with x, 0 as limit

$$\text{cdf } F^{-1}(U) = \text{solve}(F(x)=U, x)$$

Complete Distribution Reference Table

Distribution	Formula	Quick Example	Notes
Bernoulli(p)	$p^x(1-p)^{1-x}$	$p=0.75, U=0.25 \rightarrow 0.46$	Single success/failure
Discrete Unif(L,U)	$1/(U-L+1)$	$p=0.5, U=0.25 \rightarrow 0.46$	Equal probability for all values
Geometric(p)	$(1-p)^{x-1}p$	$p=0.3, U=0.72 \rightarrow 0.46$	Tries until first success
Poisson(λ)	$\frac{\lambda^x e^{-\lambda}}{x!}$	$\lambda=2, U=0.313 \rightarrow 0.46$	# of events in interval
Exponential(λ)	$\lambda e^{-\lambda x}$	$\lambda=2, U=0.3 \rightarrow 0.46$	Time between events
Uniform(a,b)	$1/(b-a)$	$a=2, b=4, U=0.25 \rightarrow 0.46$	All values equally likely
Normal(μ, σ^2)	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$\mu=5, \sigma=2, U=0.344 \rightarrow 0.46$	Real world distribution
Erlang(k, λ)	$\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$	$\lambda=2, k=3, U=0.28 \rightarrow 0.46$	Sum of k exponentials

1. REMEMBER:
• For discrete: First created x where $F(x) \geq U$
• For continuous: Use inverse CDF formula
• Build CDF tables for common discrete distributions
• Box-Muller generates TWO Normal(0,1) values from TWO Uniform(0,1) values

Box Muller Method

$$Z1 = \sqrt{-2 \cdot \ln(U1)} \cdot \cos(2\pi \cdot U2)$$

$$Z2 = \sqrt{-2 \cdot \ln(U1)} \cdot \sin(2\pi \cdot U2)$$

$$Z1/Z2 = \cot(2\pi \cdot U2) \sim \text{Cauchy}$$

$$Z2/Z1 = \tan(2\pi \cdot U2) \sim \text{Cauchy}$$

Radian-Modus einschalten!

Chi-Square Distribution

If $Z1, Z2, Z3$ are i.i.d. $\text{Nor}(0,1)$, find c such that $P(Z1^2 + Z2^2 + Z3^2 < c) = 0.99$

$$\text{Calculator: } \text{chiSqInv}(0.99, 3) \rightarrow 11.34$$

$X1, X2, X3, X4$ are i.i.d. $\text{Nor}(0,1)$. Find $\text{Pr}(X1 + X2 - X3 - X4 > 10)$

$$\text{Calculator: } \text{normCdf}(10, \text{unendlich}, 0, n/2)$$



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Find distribution of U_1 and U_2

Find distribution of $-4(U_1 + U_2) - 2$

$U_1 + U_2 \sim \text{Tria}(0, 1, 2)$

Apply the transformation $-4(U_1 + U_2) = 4(\text{Tria}(0, 1, 2))$

The minimum becomes: $-4(2) = -8$; The mode becomes: $-4(1) = -4$; The maximum becomes: $-4(0) = 0$

$4 \cdot \text{Tria}(0, 1, 2) = \text{Tria}(-8, -4, 0)$

Subtract 2

$\text{Tria}(-10, -6, -2)$

Acceptance-Rejection

Goal Generate random samples from a hard-to-sample distribution ($f(x)$).

Idea Sample from an easier distribution ($h(x)$), then accept or reject each sample based on how well it fits $f(x)$.

Example If a random variable X has the beta distribution, then its p.d.f. is of the form $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$, $0 < x < 1$, for parameters α and $\beta > 0$, and where $\Gamma(\cdot)$ is the gamma function. How might you generate such a random variate? Pick the best answer.

Goodness of fit

Find critical value Inverse χ^2 ; Area = $1-\alpha$; df = $n-1$; $\text{inv}\chi^2(0.9, 3)$

If critical value is bigger than accept H_0

Hypothesis Testing Errors

Type I Error (False positive) $\alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true})$, Fire alarm when no fire

Type II Error (False negative) $\beta = P(\text{Fail to reject } H_0 \mid H_0 \text{ is false})$; No fire alarm when there IS a fire



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