

Arena Templates

Basic Process		Advanced Process	
Module	Sheet	Module	Sheet
Create	Attribute	Seize	Adv. Set
Dispose	Entity	Delay	Expression
Process	Queue	Release	Failure
Batch	Resource		
Separate	Variable		
Assign	Schedule		
Record	Set		

Blocks		Advanced Transfer	
Module	Sheet	Module	Sheet
Seize		Station	Sequence
Delay		Route	Conveyor
Release		Enter	Transporter
Queue		Leave	Distance
			Segment

Transporter uses Request/Free, requires Distance Set.
Conveyor uses Access/Exit, requires Segment Set.

Arena Variables and Function

DISC(0.3, 1, 0.8, 2, 1.0, 3)	DISC generates random discrete values based on cumulative probabilities; pair each probability with its corresponding value.
TNOW	Current simulated time
NR(Res)	Res Servers currently in service
NQ(Queue)	Number of customers in Queue
Mod.Nu- mberOut	Customers who have left the mode

messy cdf

See F(X)	Replace with Uniform(0,1)
Multiply by 3	U(0,3)
Subtract 1	U(-1,2)
Mean	$(-1+2)/2 = 1/2$

What is the mean of the random variable $3F(X)-1$?

Inverse Transformation

Given U, find Z	$\text{invNorm}(U, 0, 1)$
Given Z, find U	$\text{normCdf}(-9999, Z, 0, 1)$
Exponential(λ)	$X = -\ln(1-U)/\lambda$
Uniform(a,b)	$X = a + (b-a) \cdot U$
Weibull(a,b)	$X = a * (-\ln(U))^{1/b}$

Inverse Transformation (cont)

Triangular	If $U < 0.5$: $\sqrt{2U}$; If $U \geq 0.5$: $2 - \sqrt{2(1-U)}$
Bernoulli(p)	If $U < 1-p \rightarrow 0$; Else $\rightarrow 1$
Poisson(λ)	Build CDF, match U
Discrete Unif(1,n)	$[n \cdot U]$
Erlang(k, λ)	$-(1/\lambda) \ln(\prod U_i)$
Geometric	$\ln(1-U)/\ln(1-p)$

For discrete: Find smallest x where $F(x) \geq U$
For continuous: Use inverse CDF formulas
Box-Muller generates TWO Normal(0,1) values from TWO Uniform(0,1) values

XOR

XOR is only true if different

Expected Value, Variance, ...

Discrete E[X]	$\text{SUM}[x * f(x)]$
Continuous E[X]	$\text{SUM}[x * f(x) dx]$
Variance of X	$E[X^2] - (E[X])^2$
Standard Deviation of X	$\text{SQRT}[\text{Var}(X)]$

Random Number Generators

Bad generators	Midsquare number generator, Random number tables, von Neumann's mid-square method, Fibonacci generator, Additive congruential generator, RANDU
Good generators	Linear Congruential Generators (modern cycle length $> 2^{191}$; Mersenne Twister (2^{19937}))
Randu	$65539X_i \text{ mod}(2^{31})$
Desert island	$16807X_i \text{ mod}(2^{31}-1) \text{ mod}(21-47483647)$
Desert island technique	$Z = [\text{SUM}(U_i) - n/2] / [\text{SQRT}(n * 1/12)]$

Inverse Transform Method Key Problems

If $X \sim \text{Normal}(0,1)$, what's the distribution of $\Phi(X)$?	Unif(0,1)
If $U \sim \text{Uniform}(0,1)$ and $\Phi(x)$ is the CDF of the standard normal, what is the distribution of $2\Phi^{-1}(U)+3$?	$\Phi^{-1}(U)$ turns a Uniform(0,1) into a Normal(0,1). Multiply by 2 $\rightarrow$ scales the standard deviation by 2 = Normal(0,4). Add 3 $\rightarrow$ shifts the mean to 3. = Normal(3,4)
$-3\ln(U^2V^2)$ where $U, V \sim \text{i.i.d. Unif}(0,1)$	$= -6\ln(U) - 6\ln(V) \sim \text{Exp}(1/6) + \text{Exp}(1/6) \sim \text{Erlang}_2(1/6)$

Joint Probability Mass Function

$E[XY]$ Summe von $x * y * f(xy)$

Joint p.m.f.

Are independent if	X and Y are independent if and only if $p_{X,Y}(x,y) = p_X(x) \cdot p_Y(y)$
For example	$P(X=1, Y=0) = P(Y=0) * P(X=1)$
$E[XY]$	Example: $(1)(0)(0.2) + (1)(1)-(0.0.0) + \dots$
$\text{Cov}(X,Y)$	$E[XY] - E[Y]E[X]$
Variance $X + Y$	$\text{Var}(X) + \text{Var}(Y) + 2\text{Cov}(X,Y)$
Variance $X - Y$	$\text{Var}(X) + \text{Var}(Y) - 2\text{Cov}(X,Y)$
Theorem	$\text{Cov}(X,Y) = 0$ if X, Y independent. Converse not true.
Correlation	$\rho = \text{Cov}(X,Y) / \text{SQRT}(\text{Var}(X) * \text{Var}(Y))$

pdf, cdf

pdf -> cdf integrate with x, 0 as limit

cdf F-1(U) solve(F(x)=U, x)

Given CDF with two cases, generate X

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F(x) = 1 - (1/2)x^20, if x < 0; 1 - (1/2)x^20, if x > 0
For X < 0 (X < 0):
  Set F(x) = (1/2)x^20 = 0
  Solve: x = -(1/2)x^20
For X > 0 (X > 0):
  Set F(x) = 1 - (1/2)x^20 = 0
  Solve: x = (1/2)x^20
Final Answer:
  x = 1 - (1/2)x^20, if x < 0; 1 - (1/2)x^20, if x > 0

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Complete Distribution Reference Table

Distribution	Formula	Quick Example	Note
Binomial(n, p)	$F(x) = 1 - \sum_{k=0}^{x-1} \binom{n-1}{k} p^k (1-p)^{n-1-k}$	$p=0.5, n=10, x=5 \rightarrow 0.5$	Single successful trial
Discrete Uniform(1, n)	$F(x) = x/n$	$n=10, x=5 \rightarrow 0.5$	Equal probability for all values
Continuous Uniform(a, b)	$F(x) = (x-a)/(b-a)$	$a=0, b=10, x=5 \rightarrow 0.5$	Equal probability for all values
Poisson(λ)	Build CDF, match U	$\lambda=2, x=3 \rightarrow 0.5$	# of events in interval
Exponential(λ)	$F(x) = 1 - e^{-\lambda x}$	$\lambda=2, x=3 \rightarrow 0.5$	Time between events
Uniform(0, 1)	$F(x) = x$	$x=0.5 \rightarrow 0.5$	# of values equally likely
Normal(μ, σ)	$F(x) = \Phi((x-\mu)/\sigma)$	$\mu=0, \sigma=1, x=0 \rightarrow 0.5$	Best curve distribution
Gamma(k, λ)	$F(x) = 1 - \sum_{i=0}^{k-1} \frac{e^{-\lambda x} (\lambda x)^i}{i!}$	$k=2, \lambda=1, x=1 \rightarrow 0.5$	Sum of n exponentials

REMEMBER:

- For discrete: Find smallest x where F(x) > U
- For continuous: Use inverse CDF formula
- Build CDF tables for custom discrete distributions
- Box-Muller generates TWO Normal(0,1) values from TWO Uniform(0,1) values

Acceptance-Rejection

Goal Generate random samples from a hard-to-sample distribution (f(x)).

Idea Sample from an easier distribution (h(x)), then accept or reject each sample based on how well it fits f(x).

Example If a random variable X has the beta distribution, then its p.d.f. is of the form $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$, $0 < x < 1$, for parameters α and $\beta > 0$, and where $\Gamma(\cdot)$ is the gamma function. How might you generate such a random variate? Pick the best answer.

Box Muller Method

Z1 $\sqrt{-2 \cdot \ln(U_1)} \cdot \cos(2\pi \cdot U_2)$

Z2 $\sqrt{-2 \cdot \ln(U_1)} \cdot \sin(2\pi \cdot U_2)$

Z1/Z2 $\cot(2\pi \cdot U_2) \sim \text{Cauchy}$

Z2/Z1 $\tan(2\pi \cdot U_2) \sim \text{Cauchy}$

Radian-Modus einschalten!

Goodness of fit

Find critical value Inverse X2; Area = 1-alpha; df = n-1; invx2(0.9,3)

If critical value is bigger than accept H0

Chi-Square Distribution

If Z1, Z2, Z3 are i.i.d. Nor(0,1), find c such that $P(Z_1^2 + Z_2^2 + Z_3^2 < c) = 0.99$

Calculator: chiSqInv(0.99, 3) → 11.34

Find distribution of U1 and U2

Find distribution of $-4(U_1 + U_2) - 2$

$U_1 + U_2 \sim \text{Tria}(0, 1, 2)$

Apply the transformation $-4(U_1 + U_2) = 4(\text{Tria}(0, 1, 2))$

The minimum becomes: $-4(2) = -8$; The mode becomes: $-4(1) = -4$; The maximum becomes: $-4(0) = 0$

$4 \cdot \text{Tria}(0, 1, 2) = \text{Tria}(-8, -4, 0)$

Subtract 2

$\text{Tria}(-10, -6, -2)$

