OMSA Midterm Exam 2 Cheat Sheet

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Given CDF with two cases, generate X

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F(x) = { (1/2)e^{3x} if x ≤ 0; 1 - (1/2)e^{-5x} if x > 0 }
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For X ≤ 0 (U ≤ 1/2):

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Set F(X) = (1/2)e^{3X} = U
Solve: X = (1/3)ln(2U)
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For X > 0 (U > 1/2):

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Set F(X) = 1 - (1/2)e^{-5X} = U
Solve: X = -(1/5)ln(2(1-U))
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Final Answer:

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X = { (1/3)ln(2U) if U ≤ 1/2; -(1/5)ln(2(1-U)) if U > 1/2 }
```

Arena Templates

Basic Process		Advanced Process	
Module	Sheet	Module	Sheet
Create	Attribute	Seize	Adv. Set
Dispose	Entity	Delay	Expression
Process	Queue	Release	Failure
Batch	Resource		
Separate	Variable		
Assign	Schedule		
Record	Set		

Blocks		Advanced Transfer	
Module	Sheet	Module	Sheet
Seize		Station	Sequence
Delay		Route	Conveyor
Release		Enter	Transporter
Queue		Leave	Distance
			Segment

Transporter uses Request/Free, requires Distance Set. Conveyor uses Access/Exit, requires Segment Set.

Arena Variables and Function

DISC(0.3, 1, 0.8, 2, 1.0, 3)	DISC generates random discrete values based on cumulative probabilities; pair each probability with its corresponding value.
TNOW	Current simulated time
NR(Res)	Res Servers currently in service
NQ(Queue)	Number of customers in Queue
Mod.NumberOut	Customers who have left the mode

Arena Set Types

Resource, Counter, entity type, entity picture

Arena Key Modules

Assign	Give new value to an attribute
Decide	Route customers probabilistically or conditonally
Separate / Clone	Split one customer into two or more clones
Route	Move entities station to station (advanced transfer)
Enter / Leave	Usually paired together for station management
Seize - Delay - Release	equals single Process module
Queue block	Cannot connect with a process module

Example of finding X for Pois

x	$f(x) = e^{-\lambda}\lambda^x/x!$	$F(x)$	Unif(0,1)'s
0	0.8187	0.8187	$[0, 0.8187]$
1	0.1637	0.9824	$(0.8187, 0.9824]$
2	0.0164	0.9988	$(0.9824, 0.9988]$
3	0.0011	0.9999	$(0.9988, 0.9999]$
≥ 4	0.0001	1.0000	$(0.9999, 1.0000]$

Notice that I'm not very concerned with the "≥ 4" case. Anyway, suppose that I draw a PRN $U = 0.726$. What value of X do I get?

(a) 0
(b) 0.726
(c) 1
(d) 2
(e) 3

Solution: From the table, we see that $U = 0.726$ clearly corresponds to $X = 0$, choice (a). □

Universal truths

−ln(U)	~Exponential(1)
	lnu

messy cdf

See F(X)	Replace with Uniform(0,1)
Multiply by 3	U(0,3)
Subtract 1	U(−1,2)
Mean	(-1+2)/2 = 1/2

What is the mean of the random variable 3F(X)−1?

Inverse Transformation

Given U, find Z	invNorm(U, 0, 1)
Given Z, find U	normCdf(-9999, Z, 0, 1)
Exponential(λ)	X = -ln(1-U)/λ
Uniform(a,b)	X = a + (b-a)·U
Weibull(a,b)	X = a * $(-\ln(U))^{1/b}$
Triangular	If U<0.5: √(2U); If U≥0.5: 2-√(2(1-U))
Bernoulli(p)	If U < 1-p → 0; Else → 1
Poisson(λ)	Build CDF, match U
Discrete Unif(1,n)	[n·U]
Erlang(k,λ)	-(1/λ)ln(∏Ui)
Geometric	ln(1-U)/ln(1-p)

For discrete: Find smallest x where F(x) ≥ U
For continuous: Use inverse CDF formulas
Box-Muller generates TWO Normal(0,1) values from TWO Uniform(0,1) values

XOR

XOR	is only true if different

Expected Value, Variance, ...

Discrete E[X]	SUM[x * f(x)]
Continous E[X]	SUM[x * f(x) dx]
Variance of X	$E[X^2] - (E[X])^2$
Standard Deviation of X	SQRT[Var(X)]

Expected Value Joint pdf

Joint p.d.f.: $f(x,y) = 2xy^2$; Domain: $0 < x < 1$, $0 < y < 1$
Find: E[2X-1]
$E[2X - 1] = \int_0^1 \int_0^1 (2x - 1) \cdot 2xy^2 \, dy \, dx$

Random Number Generators

Bad generators	Midsquare number generator, Random number tables, von Neumann's mid-square method, Fibonacci generator, Additive congruential generator, RANDU
Good generators	Linear Congruential Generators (modern cycle length > 2^{191}; Mersenne Twister (2^{19937})
Randu	65539Xi mod(2^{31})
Desert island	16807Xi mod(2^{31}-1) mod(21-47483647)
Desert island technique	Z = [SUM(Ui)-n/2] / [SQRT(n * 1/12]

Inverse Transform Method Key Problems

If X ~ Normal(0,1), what's the distribution of Φ(X)?	Unif(0,1)
If U~Uniform(0,1) and Φ(x) is the CDF of the standard normal, what is the distribution of 2Φ −1 (U)+3?	Φ−1(U) turns a Uniform(0,1) into a Normal(0,1). Multiply by 2 → scales the standard deviation by 2 = Normal(0,4). Add 3 → shifts the mean to 3. = Normal(3,4)
$-3\ell n(U^2V^2)$ where U, V ~ i.i.d. Unif(0,1)	$= -6\ell n(U) - 6\ell n(V) \sim$ Exp(1/6) + Exp(1/6) ~ $Erlang_2(1/6)$

Joint Probability Mass Function

E[XY]	Summe von x * y * f(xy)

Joint p.m.f.

Are independent if	X and Y are independent if and only if pX,Y(x,y) = pX(x) · pY(y)
For example	P(X=1, Y=0) = P(Y=0) * P(X=1)
E[XY]	Example: (1)(0)(0.2) + (1)(1)-(0.0.0) + ...
Cov(X,Y)	E[XY] - E[Y]E[X]
Variance X + Y	Var(X) + Var(Y) + 2Cov(X,Y)

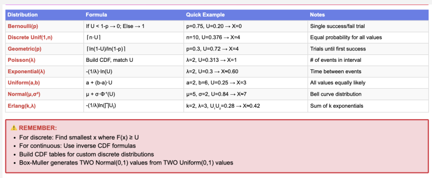
Joint p.m.f. (cont)

Variance X - Y	Var(X) + Var(Y) - 2Cov(X,Y)
Theorem	Cov(X,Y) = 0 if X, Y indepedent. Converse not true.
Correlation	p = Cov(X,Y) / SQRT(Var(X) * Var(Y))

pdf, cdf

pdf -> cdf	integrate with x, 0 as limit
cdf F-1(U)	solve(F(x)=U, x)

Complete Distribution Reference Table



Distribution	Formula	Quick Example	Notes
Bernoulli(p)	If U < 1-p → 0; Else → 1	p=0.75, U=0.20 → X=0	Single success/fail trial
Discrete Unif(1,n)	⌈n·U⌉	n=10, U=0.376 → X=4	Equal probability for all values
Geometric(p)	⌈ln(1-U)/ln(1-p)⌉	p=0.3, U=0.72 → X=4	Trials until first success
Poisson(λ)	Build CDF, match U	λ=2, U=0.313 → X=1	# of events in interval
Exponential(λ)	-(1/λ)·ln(U)	λ=2, U=0.3 → X≈0.60	Time between events
Uniform(a,b)	a + (b-a)·U	a=2, b=6, U=0.25 → X=3	All values equally likely
Normal(μ,σ²)	$\mu + \sigma \cdot \Phi^{-1}(U)$	μ=5, σ=2, U=0.84 → X≈7	Bell curve distribution
Erlang(k,λ)	$-(1/\lambda)\ln(\prod U_i)$	k=2, λ=3, U_1U_2=0.28 → X≈0.42	Sum of k exponentials

⚠ REMEMBER:
- For discrete: Find smallest x where F(x) ≥ U
- For continuous: Use inverse CDF formulas
- Build CDF tables for custom discrete distributions
- Box-Muller generates TWO Normal(0,1) values from TWO Uniform(0,1) values

Box Muller Method

Z1	$\sqrt{-2 \cdot \ln(U_1)} \cdot \cos(2\pi \cdot U_2)$
Z2	$\sqrt{-2 \cdot \ln(U_1)} \cdot \sin(2\pi \cdot U_2)$
Z1/Z2	$\cot(2\pi \cdot U_2)$ ~Cauchy
Z2/Z1	$\tan(2\pi \cdot U_2)$ ~Cauchy
Radian-Modus einschalten!	

Chi-Square Distribution

If Z_1, Z_2, Z_3 are i.i.d. Nor(0,1), find c such that $P(Z_1^2 + Z_2^2 + Z_3^2 < c) = 0.99$
Calculator: chiSqInv(0.99, 3) → 11.34
X_1, X_2, X_3, X_4 are i.i.d. Nor(0,1). Find $Pr(X_1 + X_2 - X_3 - X_4 > 10)$
Calculator normCdf(10,unendlich,0,n/2)

Find distribution of U1 and U2

Find distribution of $-4(U_1 + U_2) - 2$
$U_1 + U_2 \sim$ Tria(0, 1, 2)
Apply the transformation $-4(U_1 + U_2) =$ 4(Tria(0, 1, 2))
The minimum becomes: -4(2) = -8; The mode becomes: -4(1) = -4; The maximum becomes: -4(0) = 0
4·Tria(0, 1, 2) = Tria(-8, -4, 0)
Subtract 2
Tria(-10, -6, -2)

Acceptance-Rejection

Goal	Generate random samples from a hard-to-sample distribution (f(x)).
Idea	Sample from an easier distribution (h(x)), then accept or reject each sample based on how well it fits f(x).
Example	If a random variable X has the beta distribution, then its p.d.f. is of the form f(x) = Γ(α+β) Γ(α)Γ(β) xα−1(1−x)β−1, 0 <x<1, for parameters α and β >0, and where Γ(·) is the gamma function. How might you generate such a random variate? Pick the best answer.

Goodness of fit

Find critical value	Inverse X2; Area = 1-alpha; df = n-1; invx2(0.9,3)
	If critical value is bigger than accept H0

Hypothesis Testing Errors

Type I Error (False positive)	$\alpha = P(\text{Reject } H_0 \mid H_0 \text{ is true})$, Fire alarm when no fire
Type II Error (False negative)	$\beta = P(\text{Fail to reject } H_0 \mid H_0 \text{ is false})$; No fire alarm when there IS a fire