

Propositions		Propositions (cont)		Functions (cont)		Functions (cont)	
Different Ways of Expressing $p \rightarrow q$	q unless $\neg p$, q if p , q whenever p , q follows from p , p only if q , q when p , p is sufficient for q , q is necessary for p .	$p \leftrightarrow q$	if and only if. true if and only if p and q have the same truth value ex) $p = t$ $q = t$ or $p = f$ $q = f$	Injective Function (one to one)	a function f is one-to-one if and only if $f(a) \neq f(b)$ whenever $a \neq b$. each value in the range is mapped to exactly one element of domain. (each range value is mapped once). $\forall a \forall b (f(a) = f(b) \rightarrow a = b)$ or equivalently $\forall a \forall b (a \neq b \rightarrow f(a) \neq f(b))$	To show that f is injective	$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$. ex) $f(a) = f(b) \Rightarrow a = b$. ex) $f(x) = x + 3$. $f(a) = 7$, $a + 3 = 7$, $a = 4$. $f(b) = 7$. $b = 4$. $f(a) = f(b) \Rightarrow a = b$, 1 to 1
Proposition	True/False, with no variables. Ex) The sky is blue = Prop. $n + 1$ is even Not prop bc n is unknown.	Logically equivalent	$p \equiv q$. all truth values have to be the equal aka same results.	Surjective function (onto)	every element in codomain maps to at least one element in domain. (each element in codomain is mapped). if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$	To show that f is not injective	Find particular elements $x, y \in A$ such that $x \neq y$ and $f(x) = f(y)$
Tautology	a proposition which is always true. Ex) $p \vee \neg p$	Negate Quantifiers	$\neg \forall x P(x) \equiv \exists x \neg P(x)$. $\neg \exists x Q(x) \equiv \forall x \neg Q(x)$	domain of f	the set A , where f is a function from A to B . ans is A	To show that f is surjective	solve in terms of x . pick 2 random y s, if x eqns comes back in domain, surjective. domain matters, Z, R, N has to map x and y in same. ex) $f(x) = x + 3$. $f(4) = 7$, $f(5) = 8$. always mapped, onto.
contradiction	a proposition which is always false. Ex) $p \wedge \neg p$	Functions	function from A to B : $f: A \rightarrow B$	codomain of f	the set B , where f is a function from A to B . ans is B	To show that f is not surjective	Find a particular $y \in B$ such that $f(x) \neq y$ for all $x \in A$
contingency	a proposition which is neither a tautology nor a contradiction, such as p	b is the image of a under f	$b = f(a)$. "what does this map to"	a is a pre-image of b under f	$f(a) = b$. "what values map to this".	Bijective function	all range is mapped to and mapped to once (injective and surjective)
satisfiable	at least one truth table is true.	range	values of codomain that were mapped to by domain.				
$p \rightarrow q$	Only false when $p = T$ $q = F$. everthing else true.						
converse	$q \rightarrow p$						
inverse	$\neg p \rightarrow \neg q$						
contrapositive	$\neg q \rightarrow \neg p$						



Functions (cont)		Functions (cont)		Proofs (cont)		Proofs (cont)	
Inverse	has to be bijective. $f^{-1}(y) = x$ if and only if $f(x) = y$. because this is both 1to1 and onto, its a bijection, therefore invertible.	ex) let $f(0) = 0, f(1) = 0, f(2) = 2, f(3) = 4$	$f(x) = \text{floor}((x^2 - 2)/2)$. find $f(S)$ if $S = \{0, 1, 2, 3\}$	Proof by contradiction	Assume p is true, find contradiction, therefore p is true. prove that p is true if we can show that $\neg p \rightarrow (r \wedge \neg r)$ is true for some proposition r	UNIQUENESS proof	When asked for unique, prove exists, then unique. ex: $x \neq y$, so y doesnt have that property, therefore x is unique.
composition of fns	$f(g(a))$ or $f \circ g(a)$	equal functions	Two functions are equal when they have the same domain, the same codomain and map each element of the domain to the same element of the codomain	Counterexample	to show that a statement of the form $\forall x P(x)$ is false, we need only find a counterexample.	without loss of generality	an assumption in a proof that makes it possible to prove a theorem by reducing the number of cases to consider in the proof
floor/ceiling	bracketwithlow only/highonly. round down/up to nearest integer. ex) $-2.2 \text{ floor} = -3, 5.5 \text{ ceil} = 6$.			Proof by exhaustion	ex: Prove that $(n + 1)^3 \geq 3n$ if n is a positive integer with $n \leq 4$. Prove by doing $n = 1, 2, 3, 4$		
properties	$x \text{ floor} = n$ if and only if $n \leq x < n + 1$. $x \text{ ceil} = n$ if and only if $n - 1 < x \leq n$. $x \text{ floor} = n$ if and only if $x - 1 < n \leq x$. $x \text{ ceil} = n$ if and only if $x \leq n < x + 1$. $x - 1 < \text{floor} x \leq x \leq \text{ceil} x < x + 1$. $\text{floor} x = -\text{ceil} x$. $\text{ceil} x = -\text{floor} x$. $\text{ceil}(x + n) = \text{ceil} x + n$. opp of last floor	Proofs		Proof by cases	ex: Prove that if n is an integer, then $n^2 \geq n$. Case (i): When $n = 0$. Case (ii): When $n \geq 1$. Case (iii): In this case $n \leq -1$	Sets	
		Direct Proof	assume p is true, prove q . $p \Rightarrow q$. Always start with this then try contraposition.	Constructive Existence Proof	$\exists x P(x)$. To find if $P(x)$ exists, show an example $P(c) = \text{True}$	Element of set	$a \in A, a \notin A$
		Proof by contraposition	assume q is true, prove $\neg p$. $(q \Rightarrow \neg p)$ equals $(p \Rightarrow q)$	Nonconstructive Existence Proof	Assume no values makes $P(x)$ true. Then contradict.	roster method	$V = \{a, e, i, o, u\}, O = \{1, 3, 5, 7, 9\}$.
		Vacuous proof	if we can show that p is false, then we have a proof, called a vacuous proof, of the conditional statement $p \rightarrow q$			set builder notation	ex: the set O of all odd positive integers less than 10 can be written as: $O = \{x \in \mathbb{Z}^+ \mid x \text{ is odd and } x < 10\}$. ex) $A = \{x \mid x \geq -1 \wedge x < 1\}$. ex) $\{x \mid P(x)\}$
						Interval Notation	$[-2, 8)$
						Natural numbers	$\mathbb{N} = \{0, 1, 2, 3, \dots\}$
						\mathbb{N}	



Sets (cont)		Sets (cont)		Sets (cont)		Sets (cont)	
Integers Z	$Z = \{ \dots, -2, -1, 0, 1, 2, \dots \}$	Universal Set U	Universe in context of statement. Example vowels in alphabet: $U = \{z,y,x,w, \dots\}$, $A = \{a,e,i,o,u\}$ A is a subset of U.	proper subset	$\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A) A \subset B$ but $A \neq B$. B contains an element not in A. Ex) $A = \{1,2,3\}$, $B = \{1,2,3,4\}$. 4 makes it proper subset.	Cartesian Product	$\{(a, b) \mid a \in A \wedge b \in B\}$. The Cartesian product of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Ex) $A = \{0,1\}$ $B = \{2,3,4\}$, $A \times B = \{(0,2),(0,3),(0,4), (1,2),(1,3),(1,4)\}$
Positive Integers Z^+	$Z^+ = \{1, 2, 3, \dots\}$	Subset	$\forall x(x \in A \rightarrow x \in B)$. Ex) the set A is a subset of B if and only if every element of A is also an element of B. We use the notation $A \subseteq B$ to indicate that A is a subset of the set B. Ex) $A = \{1,2,3\}$, $B = \{1,2,3,4\}$, $A \subseteq B$.	Cardinality	$ A $ Distinct elements of set. $A = \{1,2,3,3,4,4\}$ $ A = 4$	Truth Set	$P(x): \text{abs}(x) = 3$. Truth Set of $P(x) = \{3,-3\}$
Rational Numbers Q	$Q = \{p/q \mid p \in Z, q \in Z, \text{ and } q \neq 0\}$	Showing that A is a Subset of B	To show that $A \subseteq B$, show that if x belongs to A then x also belongs to B.	Power Set	the power set of S is the set of all subsets of the set S. The power set of S is denoted by $P(S)$. Ex) $A = \{1,2,3\}$. $P(A) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$. Ex) $P(\{\emptyset\}) = \{\emptyset, \{\emptyset\}\}$	Set Operations	
Real Numbers R	All previous sets (N, Z, Q)	Showing that A is Not a Subset of B	To show that $A \not\subseteq B$, find a single $x \in A$ such that $x \notin B$.	Cardinality of Power Set	2^n , n is elements.	Union	$A \cup B = \{x \mid x \in A \vee x \in B\}$. Ex) $A = \{1,4,7\}$ $B = \{4,5,6\}$. $A \cup B = \{1,4,5,6,7\}$
R^+	positive real numbers	Showing Two Sets are Equal	To see if $A = B$, Show $A \subseteq B$ and $B \subseteq A$	Tuple	$(a_1, a_2, a_3, \dots, a_n)$ Ordered. Ex) $(5,2) \neq (2,5)$	Intersection	$A \cap B = \{x \mid x \in A \wedge x \in B\}$. Ex) $A = \{1,4,7\}$ $B = \{4,5,6\}$. $A \cap B = \{4\}$.
Complex numbers C	$\{a+bi, \dots\}$					disjoint	If $A \cap B = \text{nothing}$, A and B are disjoint.
Equal Sets	Two sets are equal if and only if they have the same elements. Therefore, if A and B are sets, then A and B are equal if and only if $\forall x(x \in A \leftrightarrow x \in B)$. We write $A = B$ if A and B are equal sets. Dont matter if its $\{1,3,3,3,2,2,3\}$, still $\{1,3,2\}$. Also dont matter order.						
Null/Empty Set	\emptyset , nothing. $\{\}$.						
$\{\emptyset\}$	1 element						
Singleton set	One element.						



Set Operations (cont)

principle of inclusion-exclusion
 $|A \cup B| = |A| + |B| - |A \cap B|$
 ex) $A = \{1,2,3,4,5\}$, $B = \{4,5,6,7,8\}$.
 $|A \cup B| = 5 + 5 - 2 = 8$

$A - B$, difference of A and B
 $A - B = A \cap B^c = \{x \mid x \in A \wedge x \notin B\}$
 Elements in A that are not in B.
 Ex) $\{1, 3, 5\} - \{1, 2, 3\} = \{5\}$. This is different from the difference of $\{1, 2, 3\}$ and $\{1, 3, 5\}$, which is the set $\{2\}$.

Complement of A, A^c
 $\{x \in U \mid x \notin A\}$
 Everything in the universe context that's not in A.
 Ex) $U = \{1,2,3,4\}$.
 $A = \{2\}$ $B = \{3\}$.
 $A^c = \{1,3,4\}$

$U = \mathbb{R}$
 $A = \{x \mid x \geq -1 \wedge x < 1\}$
 $B = \{x \mid x < 0 \vee x \geq 2\}$
 $A \cup B = \{x \mid \square < 1 \vee x \geq 2\}$.
 $A \cap B = \{x \mid x < 0 \wedge \square \geq -1\}$.
 $A^c = \{x \mid x < -1 \vee x \geq 1\}$.

Set Operations (cont)

Identity, , , , , , absorption,
 $A \cap U = A$. $A \cup \emptyset = A$.

domination
 $A \cup U = U$. $A \cap \emptyset = \emptyset$

idempotent
 $A \cup A = A$. $A \cap A = A$

complementation
 $(A^c)^c = A$

commutative
 $A \cup B = B \cup A$. $A \cap B = B \cap A$

associative
 $A \cup (B \cup C) = (A \cup B) \cup C$. $A \cap (B \cap C) = (A \cap B) \cap C$

distributive
 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
 $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

de morgan's
 $(A \cap B)^c = A^c \cup B^c$. $(A \cup B)^c = A^c \cap B^c$

absorption
 $A \cup (A \cap B) = A$. $A \cap (A \cup B) = A$.

complement
 $A \cup A^c = U$. $A \cap A^c = \emptyset$.

Set Operations (cont)

countable
 a set that either is finite or can be placed in one-to-one correspondence with the set of positive integers. To be countable, there must exist a 1-1 and onto (bijection) between the set and $\mathbb{N}!$ (i.e. \mathbb{Z}^+)

Ex) Show that the set of positive even integers E is countable set.
 Let $f(x) = 2x$. Then f is a bijection from \mathbb{N} to E since f is both one-to-one and onto. To show that it is one-to-one, suppose that $f(n) = f(m)$. Then $2n = 2m$, and so $n = m$. To see that it is onto, suppose that t is an even positive integer. Then $t = 2k$ for some positive integer k and $f(k) = t$