Propositions

Different Ways of Expressing p → q	q unless ¬p, q if p, q whenever p, q follows from p, p only if q, q when p, p is sufficient for q, q is necessary for p.
Proposition	True/False, with no variables. Ex) The sky is blue = Prop. n+1 is even Not prop bc n is unknown.
Tautology	a proposition which is always true. Ex) p v¬ p
contradiction	a proposition which is always false. Ex) p ∧¬ p
contingency	a proposition which is neither a tautology nor a contradiction, such as p
satisfiable	at least one truth table is true.
p -> q	Only false when p = T q = F. everthing else true.
converse	q -> p
inverse	-p -> -q
contrapositive	-q -> -p

Propositions (cont)

p <-> q	if and only if. true if and only if p and q have the same truth value ex) p = t q = t or p = f q = f
Logically equivalent	p≡q. all truth values have to be the equal aka same results.
Negate Quantifiers	¬∀xP (x) ≡ ∃x ¬P (x). ¬∃xQ(x) ≡ ∀x ¬Q(x)

Functions

function from A to B. f: A -> B	an assignment of exactly one element of B to each element of A
domain of f	the set A, where f is a function from A to B. ans is A
codomain of f	the set B, where f is a function from A to B. ans is B
b is the image of a under f	b = f (a). "what does this map to"
a is a pre-image of b under f	f (a) = b. "what values map to this".
range	values of codomain that were mapped to by domain.

Functions (cont)

Injective Function (one to one)	a function f is one-to-one if and only if f (a) =/= f (b) whenever a =/= b. each value in the range is mapped to exactly one element of domain. (each range value is mapped once). ∀a∀b(f (a) = f (b) → a = b) or equivalently ∀a∀b(a =/= b → f (a) =/= f (b))
Surjective function (onto)	every element in codomain maps to at least one element in domain. (each element in codomain is mapped). if and only if for every element b ∈ B there is an element a ∈ A with f (a) = b

Functions (cont)

To show that f is injective	f(x1)=f(x2) => x1=x2. x1=/=x2 => f(x1)=/=f(x2). ex) f(a) = f(b) => a=b. ex) f(x) = x+3. f(a) = 7, a+3=7, a=4. f(b)=7. b=4. f(a)=f(b) a=b, 1to1
To show that f is not injective	Find particular elements x, y ∈ A such that x /= y and f (x) = f (y)
To show that f is surjective	solve in terms of x. pick 2 random ys, if x eqns comes back in domain, surjective. domain matters, Z, R, N has to map x and y in same. ex) f(x)=x+3. f(4)=7, f(5)=8. always mapped, onto.
To show that f is not surjective	Find a particular y ∈ B such that f (x) = y for all x ∈ A
Bijective function	all range is mapped to and mapped to once (injective and surjective)

Functions (cont)

Inverse	has to be bijective. f^-1(y) = x if and only if f(x) = y. because this is both 1to1 and onto, its a bijection, therefore invertible.
compos-ition of fns	f(g(a)) or f o g(a)
floor/-ceiling	bracketwithlow only/highonly. round down/up to nearest integer. ex) -2.2 floor = -3. 5.5 ceil = 6.
properties	x floor = n if and only if n ≤ x < n + 1. x ciel = n if and only if n − 1 < x ≤ n. x floor = n if and only if x − 1 < n ≤ x. x ciel = n if and only if x ≤ n < x + 1. x − 1 < floorx ≤ x ≤ cielx < x + 1. floorx = −cielx. cielx = -floorx. ciel(x + n) = cielx + n. opp of last floor

Functions (cont)

ex) let f(x) = floor((x^-2)/2). find f(S) if S= {0,1,2,3}	f(0) = 0, f(1) = 0. f(2) = 2, f(3) = 4
equal functions	Two functions are equal when they have the same domain, the same codomain and map each element of the domain to the same element of the codomain

Proofs

Direct Proof	assume p is true, prove q. p => q. Always start with this then try contrapos-ition.
Proof by contra-position	assume ~q is true, prove ~p. (~q => ~p) equals (p => q)
Vacuous proof	if we can show that p is false, then we have a proof, called a vacuous proof, of the conditional statement p → q

Proofs (cont)

Proof by contra-diction	Assume ~p is true, find contradiction, therefore ~p is true. prove that p is true if we can show that ¬p → (r ∧ ¬r) is true for some proposition r
Counte-rexample	to show that a statement of the form ∀xP (x) is false, we need only find a counte-rexample.
Proof by exhuastion	ex: Prove that (n + 1)3 ≥ 3n if n is a positive integer with n ≤ 4. Prove by doing n = 1,2,3,4
Proof by cases	ex: Prove that if n is an integer, then n2 ≥ n. Case (i): When n = 0. Case (ii): When n ≥ 1. Case (iii): In this case n ≤ −1
Constr-uctive Existence Proof	∃xP(x). To find if P(x) exists, show an example P(c) = True
Noncon-structive Existence Proof	Assume no values makes P(x) true. Then contradict.

Proofs (cont)

UNIQUENESS proof	When asked for unique, prove exists, then unique. ex: x exists, x=/=y , so y doesnt have that property, therefore x is unique.
without loss of generality	an assumption in a proof that makes it possible to prove a theorem by reducing the number of cases to consider in the proof

Sets

Element of set	a ∈ A, a ∈/ A
roster method	V = {a, e, i, o, u}, O = {1, 3, 5, 7, 9}.
set builder notation	ex: the set O of all odd positive integers less than 10 can be written as: O = {x ∈ Z+ | x is odd and x < 10}. ex) A = {x|x ≥ −1 ∧ x < 1}. ex) {x | P(x)|
Interval Notation	[-2,8)
Natural numbers N	N = {0, 1, 2, 3, . . .}

Sets (cont)

Integers Z	Z = {. . . , −2, −1, 0, 1, 2, . . .}
Positive Integers Z+	Z+ = {1, 2, 3, . . .}
Rational Numbers Q	Q = {p/q | p ∈ Z, q ∈ Z, and q =/= 0}
Real Numbers R	All previous sets (N, Z, Q)
R+	positive real numbers
Complex numbers C	{a+bi, ...}
Equal Sets	Two sets are equal if and only if they have the same elements. Therefore, if A and B are sets, then A and B are equal if and only if ∀x(x ∈ A ↔ x ∈ B). We write A = B if A and B are equal sets. Dont matter if its {1,3,3,3,2,2,3,}, still {1,3,2}. Also dont matter order.
Null/ Empty Set	∅, nothing. {}.
{∅}	1 element
Singleton set	One element.

Sets (cont)

Universal Set U	Universe in context of statement. Example vowels in alphabet: U = {z,y,x,w, ...}, A = {a,e,i,o,u} A is a subset of U.
Subset	∀x(x ∈ A → x ∈ B). Ex) the set A is a subset of B if and only if every element of A is also an element of B. We use the notation A ⊆ B to indicate that A is a subset of the set B. Ex) A = {1,2,3}, B = {1,2,3,4}, A ⊆ B.
Showing that A is a Subset of B	To show that A ⊆ B, show that if x belongs to A then x also belongs to B.
Showing that A is Not a Subset of B	To show that A ⊆/ B, find a single x ∈ A such that x ∈/ B.
Showing Two Sets are Equal	To see if A = B, Show A ⊆ B and B ⊆ A

Sets (cont)

proper subset	∀x(x ∈ A → x ∈ B) ∧ ∃x(x ∈ B ∧ x ∈ A)A ⊆ B but A =/= B. B contains an element not in A. Ex) A = {1,2,3}, B= {1,2,3,4}. 4 makes it proper subset.
Cardin ality	|A| Distinct elements of set. A = {1,2,3,3,4,4} |A| = 4
Power Set	the power set of S is the set of all subsets of the set S. The power set of S is denoted by P(S). Ex) A = {1,2,3}. P(A) = {∅, {0}, {1}, {2}, {0, 1}, {0, 2}, {1, 2}, {0, 1, 2}}. Ex) P({∅}) = {∅, {∅}}
Cardin ality of Power Set	2^n, n is elements.
Tuple	(a1,a2,a3, ..., an) Ordered. Ex) (5,2) =/= (2,5)

Sets (cont)

Cartesian Product	{(a, b) | a ∈ A ∧ b ∈ B}. The Cartesian product of A and B, denoted by A × B, is the set of all ordered pairs (a, b), where a ∈ A and b ∈ B. Ex) A = {0,1} B = {2,3,4}, A x B = {(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)}
Truth Set	P(x): abs(x) = 3. Truth Set of P(x) = {3,-3}

Set Operations

Union	A ∪ B = {x | x ∈ A ∨ x ∈ B}. Ex) A = {1,4,7} B = {4,5,6}. A ∪ B = {1,4,5,6,7}
Inters-ection	A ∩ B = {x | x ∈ A ∧ x ∈ B}. Ex) A = {1,4,7} B = {4,5,6}. A ∩ B = {4}.
disjoint	If A ∩ B = nothing, A and B are disjoint.

Set Operations (cont)

principle of inclusion–exclusion $\|A \cup B\| = \|A\| + \|B\| - \|A \cap B\|$	ex) A = {1,2,3,4,5}, B = {4,5,6,7,8}. }A u B| = |A| + |B| -|A n B} = 5 + 5 - 2 = 8
A − B, difference of A and B	A - B = A ∩ B. {x | x ∈ A ∧ x /∈ B} Elements in A that are not in B. Ex) {1, 3, 5} − {1, 2, 3} = {5}. This is different from the difference of {1, 2, 3} and {1, 3, 5}, which is the set {2}.
Complement of A, A^c	{x ∈ U | x /∈ A} Everything in the universe context thats not in A. Ex) U = {1,2,3,4}. A = {2} B = {3}. A^c = {1,3,4}
U = ℝ A = {𝑥|𝑥 ≥ −1 ∧ 𝑥 < 1}, B ={𝑥|𝑥 < 0 ∨ 𝑥 ≥ 2}	𝐴 ∪ 𝐵 = {𝑥|☐ ☐ < 1 ∨ 𝑥 ≥ 2}. 𝐴 ∩ 𝐵 = {𝑥|𝑥 < 0 ∧ ☐ ☐ ≥ −1}. 𝐴^c = {𝑥|𝑥 < −1 ∨ 𝑥 ≥ 1}.

Set Operations (cont)

Identity, , , , , , , absorption,	A ∩ U = A. A ∪ ∅ = A.
domination	A ∪ U = U. A ∩ ∅ = ∅
idempotent	A ∪ A = A. A ∩ A = A
complementation	$(A^{c)}c = A$
commutative	A ∪ B = B ∪ A. A ∩ B = B ∩ A
associative	A ∪ (B ∪ C) = (A ∪ B) ∪ C. A ∩ (B ∩ C) = (A ∩ B) ∩ C
distributive	A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C). A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)
de morgans	$(A n b)^{c = A}c u B^{c. (A U B)}c = A^{c n B}c$
absorption	A ∪ (A ∩ B) = A. A ∩ (A ∪ B) = A.
complement	$A \cup A^{c = U. A \cap A}c = \varnothing$.

Set Operations (cont)

countable	a set that either is finite or can be placed in one-to-one correspondence with the set of positive integers. To be countable, there must exist a 1-1 and onto (bijection) between the set and ℕ! (i.e. ℤ+)
Ex) Show that the set of positive even integers E is countable set.	Let f(x) = 2x. Then f is a bijection from N to E since f is both one-to-one and onto. To show that it is one-to-one, suppose that f(n) = f(m). Then 2 n = 2 m, and so n = m. To see that it is onto, suppose that t is an even positive integer. Then t = 2k for some positive integer k and f(k) = t