## Propositions

| | |
|---|---|
| Different Ways of Expressing p → q | q unless ¬p, q if p, q whenever p, q follows from p, p only if q, q when p, p is sufficient for q, q is necessary for p. |
| Propos-ition | True/False, with no variables. Ex) The sky is blue = Prop. n+1 is even Not prop bc n is unknown. |
| Tautology | a proposition which is always true. Ex) p ∨¬ p |
| contra-diction | a proposition which is always false. Ex) p ∧¬ p |
| contin-gency | a proposition which is neither a tautology nor a contradiction, such as p |
| satisfiable | at least one truth table is true. |
| p -> q | Only false when p = T q = F. everthing else true. |
| converse | q -> p |
| inverse | -p -> -q |
| contrapos-itive | -q -> -p |

## Propositions (cont)

| | |
|---|---|
| p <-> q | if and only if. true if and only if p and q have the same truth value ex) p = t q = t or p = f q = f |
| Logically equivalent | p≡q. all truth values have to be the equal aka same results. |
| Negate Quanti-fiers | ¬∀xP (x) ≡ ∃x ¬P (x). ¬∃xQ(x) ≡ ∀x ¬Q(x) |

## Functions

| | |
|---|---|
| function from A to B. f: A -> B | an assignment of exactly one element of B to each element of A |
| domain of f | the set A, where f is a function from A to B. ans is A |
| codomain of f | the set B, where f is a function from A to B. ans is B |
| b is the image of a under f | b = f (a). "what does this map to" |
| a is a pre-image of b under f | f (a) = b. "what values map to this". |
| range | values of codomain that were mapped to by domain. |

## Functions (cont)

| | |
|---|---|
| Injective Function (one to one) | a function f is one-to-one if and only if f (a) =/= f (b) whenever a =/= b. each value in the range is mapped to exactly one element of domain. (each range value is mapped once). ∀a∀b(f (a) = f (b) → a = b) or equiva-lently ∀a∀b(a =/= b → f (a) =/= f (b)) |
| Surjective function (onto) | every element in codomain maps to at least one element in domain. (each element in codomain is mapped). if and only if for every element b ∈ B there is an element a ∈ A with f (a) = b |

## Functions (cont)

| | |
|---|---|
| To show that f is injective | f(x1)=f(x2) => x1=x2. x1=/=x2 => f(x1)=/=f(x2). ex) f(a) = f(b) => a=b. ex) f(x) = x+3. f(a) = 7, a+3=7, a=4. f(b)=7. b=4. f(a)=f(b) a=b, 1to1 |
| To show that f is not injective | Find particular elements x, y ∈ A such that x /= y and f (x) = f (y) |
| To show that f is surjective | solve in terms of x. pick 2 random ys, if x eqns comes back in domain, surjec-tive. domain matters, Z, R, N has to map x and y in same. ex) f(x)=x+3. f(4)=7, f(5)=8. always mapped, onto. |
| To show that f is not surjective | Find a particular y ∈ B such that f (x) = y for all x ∈ A |
| Bijective function | all range is mapped to and mapped to once (injective and surjective) |

## Functions (cont)

| | |
|---|---|
| Inverse | has to be bijective. f^-1(y) = x if and only if f(x) = y. because this is both 1to1 and onto, its a bijection, therefore invertible. |
| compos-ition of fns | f(g(a)) or f o g(a) |
| floor/-ceiling | bracketwithlow only/highonly. round down/up to nearest integer. ex) -2.2 floor = -3. 5.5 ceil = 6. |
| properties | x floor = n if and only if n ≤ x < n + 1. x ciel = n if and only if n − 1 < x ≤ n. x floor = n if and only if x − 1 < n ≤ x. x ciel = n if and only if x ≤ n < x + 1. x − 1 < floorx ≤ x ≤ cielx < x + 1. floorx = −cielx. cielx = -floorx. ciel(x + n) = cielx + n. opp of last floor |

## Functions (cont)

| | |
|---|---|
| ex) let f(x) = floor((x^-2)/2). find f(S) if S= {0,1,2,3} | f(0) = 0, f(1) = 0. f(2) = 2, f(3) = 4 |
| equal functions | Two functions are equal when they have the same domain, the same codomain and map each element of the domain to the same element of the codomain |

## Proofs

| | |
|---|---|
| Direct Proof | assume p is true, prove q. p => q. Always start with this then try contrapos-ition. |
| Proof by contra-position | assume q is true, prove p. (q => p) equals (p => q) |
| Vacuous proof | if we can show that p is false, then we have a proof, called a vacuous proof, of the conditional statement p → q |

## Proofs (cont)

| | |
|---|---|
| Proof by contra-diction | Assume p is true, find contradiction, therefore p is true. prove that p is true if we can show that ¬p → (r ∧ ¬r) is true for some proposition r |
| Counte-rexample | to show that a statement of the form ∀xP (x) is false, we need only find a counte-rexample. |
| Proof by exhuastion | ex: Prove that (n + 1)3 ≥ 3n if n is a positive integer with n ≤ 4. Prove by doing n = 1,2,3,4 |
| Proof by cases | ex: Prove that if n is an integer, then n2 ≥ n. Case (i): When n = 0. Case (ii): When n ≥ 1. Case (iii): In this case n ≤ −1 |
| Constr-uctive Existence Proof | ∃xP(x). To find if P(x) exists, show an example P(c) = True |
| Noncon-structive Existence Proof | Assume no values makes P(x) true. Then contradict. |

## Proofs (cont)

| | |
|---|---|
| UNIQUENESS proof | When asked for unique, prove exists, then unique. ex: x exists, x=/=y , so y doesnt have that property, therefore x is unique. |
| without loss of generality | an assumption in a proof that makes it possible to prove a theorem by reducing the number of cases to consider in the proof |

## Sets

| | |
|---|---|
| Element of set | a ∊ A, a ∊/ A |
| roster method | V = {a, e, i, o, u}, O = {1, 3, 5, 7, 9}. |
| set builder notation | ex: the set O of all odd positive integers less than 10 can be written as: O = {x ∊ Z+ \| x is odd and x < 10}. ex) A = {$x$\|$x$ ≥ −1 ∧ $x$ < 1}. ex) {x \| P(x)\| |
| Interval Notation | [-2,8) |
| Natural numbers N | N = {0, 1, 2, 3, . . .} |

## Sets (cont)

| | |
|---|---|
| Integers Z | Z = {. . . , −2, −1, 0, 1, 2, . . .} |
| Positive Integers Z+ | Z+ = {1, 2, 3, . . .} |
| Rational Numbers Q | Q = {p/q \| p ∈ Z, q ∈ Z, and q =/= 0} |
| Real Numbers R | All previous sets (N, Z, Q) |
| R+ | positive real numbers |
| Complex numbers C | {a+bi, ...} |
| Equal Sets | Two sets are equal if and only if they have the same elements. Therefore, if A and B are sets, then A and B are equal if and only if ∀x(x ∈ A ↔ x ∈ B). We write A = B if A and B are equal sets. Dont matter if its {1,3,3,3,2,2,3,}, still {1,3,2}. Also dont matter order. |
| Null/ Empty Set | ∅, nothing. {}. |
| {∅} | 1 element |
| Singleton set | One element. |

## Sets (cont)

| | |
|---|---|
| Universal Set U | Universe in context of statement. Example vowels in alphabet: U = {z,y,x,w, ...}, A = {a,e,i,o,u} A is a subset of U. |
| Subset | ∀x(x ∈ A → x ∈ B). Ex) the set A is a subset of B if and only if every element of A is also an element of B. We use the notation A ⊆ B to indicate that A is a subset of the set B. Ex) A = {1,2,3}, B = {1,2,3,4}, A ⊆ B. |
| Showing that A is a Subset of B | To show that A ⊆ B, show that if x belongs to A then x also belongs to B. |
| Showing that A is Not a Subset of B | To show that A ⊆/ B, find a single x ∈ A such that x ∈/ B. |
| Showing Two Sets are Equal | To see if A = B, Show A ⊆ B and B ⊆ A |

## Sets (cont)

| | |
|---|---|
| proper subset | ∀x(x ∈ A → x ∈ B) ∧ ∃x(x ∈ B ∧ x  ∈ A)A ⊆ B but A =/= B. B contains an element not in A. Ex) A = {1,2,3}, B= {1,2,3,4}. 4 makes it proper subset. |
| Cardinality | \|A\| Distinct elements of set. A = {1,2,3,3,4,4} \|A\| = 4 |
| Power Set | the power set of S is the set of all subsets of the set S. The power set of S is denoted by P(S). Ex) A = {1,2,3}. P(A) = {∅, {0}, {1}, {2}, {0, 1}, {0, 2}, {1, 2}, {0, 1, 2}}. Ex) P({∅}) = {∅, {∅}} |
| Cardinality of Power Set | 2^n, n is elements. |
| Tuple | (a1,a2,a3, ..., an) Ordered. Ex) (5,2) =/= (2,5) |

## Sets (cont)

| | |
|---|---|
| Cartesian Product | {(a, b) \| a ∈ A ∧ b ∈ B}. The Cartesian product of A and B, denoted by A × B, is the set of all ordered pairs (a, b), where a ∈ A and b ∈ B. Ex) A = {0,1} B = {2,3,4}, A x B = {(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)} |
| Truth Set | P(x): abs(x) = 3. Truth Set of P(x) = {3,-3} |

## Set Operations

| | |
|---|---|
| Union | A ∪ B = {x \| x ∈ A ∨ x ∈ B}. Ex) A = {1,4,7} B = {4,5,6}. A ∪ B = {1,4,5,6,7} |
| Inters-ection | A ∩ B = {x \| x ∈ A ∧ x ∈ B}. Ex) A = {1,4,7} B = {4,5,6}. A ∩ B = {4}. |
| disjoint | If A ∩ B = nothing, A and B are disjoint. |

## Set Operations (cont)

| | |
|---|---|
| principle of inclusion–exclusion \|A ∪ B\| = \|A\| + \|B\| − \|A ∩ B\| | ex) A = {1,2,3,4,5}, B = {4,5,6,7,8}. }A u B\| = \|A\| + \|B\| -\|A n B} = 5 + 5 - 2 = 8 |
| A − B, difference of A and B | A - B = A ∩ B. {x \| x ∊ A ∧ x /∊ B} Elements in A that are not in B. Ex) {1, 3, 5} − {1, 2, 3} = {5}. This is different from the difference of {1, 2, 3} and {1, 3, 5}, which is the set {2}. |
| Complement of A, A^c | {x ∊ U \| x /∊ A} Everything in the universe context thats not in A. Ex) U = {1,2,3,4}. A = {2} B = {3}. A^c = {1,3,4} |
| U = ℝ A = {𝑥\|𝑥 ≥ −1 ∧ 𝑥 < 1}, B ={𝑥\|𝑥 < 0 ∨ 𝑥 ≥ 2} | 𝐴 ∪ 𝐵 = {𝑥\|☐ ☐ < 1 ∨ 𝑥 ≥ 2}. 𝐴 ∩ 𝐵 = {𝑥\|𝑥 < 0 ∧ ☐ ☐ ≥ −1}. 𝐴^c = {𝑥\|𝑥 < −1 ∨ 𝑥 ≥ 1}. |

## Set Operations (cont)

| | |
|---|---|
| Identity, , , , , , , absorbtion, | A ∩ U = A. A ∪ ∅ = A. |
| domination | A ∪ U = U. A ∩ ∅ = ∅ |
| idempotent | A ∪ A = A. A ∩ A = A |
| complementation | $(A^{c)}c$ = A |
| commutative | A ∪ B = B ∪ A. A ∩ B = B ∩ A |
| associative | A ∪ (B ∪ C) = (A ∪ B) ∪ C. A ∩ (B ∩ C) = (A ∩ B) ∩ C |
| distributive | A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C). A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C) |
| de morgans | $(A \text{ n } b)^{c = A}c$ u $B^{c. (A \cup B)}c = A^{c \text{ n } B}c$ |
| absorption | A ∪ (A ∩ B) = A. A ∩ (A ∪ B) = A. |
| complement | A ∪ $A^{c = U. A \cap A}$ c = ∅. |

## Set Operations (cont)

| | |
|---|---|
| countable | a set that either is finite or can be placed in one-to-one correspondence with the set of positive integers. To be countable, there must exist a 1-1 and onto (bijection) between the set and ℕ! (i.e. ℤ+) |
| Ex) Show that the set of positive even integers E is countable set. | Let f(x) = 2x. Then f is a bijection from N to E since f is both one-to-one and onto. To show that it is one-to-one, suppose that f( n) = f( m). Then 2 n = 2 m, and so n = m. To see that it is onto, suppose that t is an even positive integer. Then t = 2k for some positive integer k and f(k) = t |