Trigonometry Questions Year 9 Cheat Sheet

by inkirbythesecond via cheatography.com/182080/cs/37862/

RUBRICK

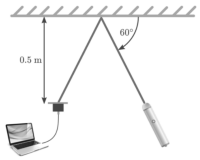
Level 7 - 8	I. select appropriate mathematics when solving **challenging problems** in both **familiar and unfamiliar situations** II. apply the selected mathematics successfully when solving these problems III. generally solve these problems correctly in a variety of contexts.	i. **consistently** use **appropriate** mathematical language (units, conclusion, notation, labels, sides and angles identified) ii. **consistently** use **appropriate forms** of mathematical representation to present information **correctly (diagrams)** iii. communicate **clearly** through **coherent** lines of reasoning (writing equations and lines of working) iv. present work that is **consistently organized** using a logical structure.

ANSWERS BELOW

QUESTION 1

You are performing a science experiment involving measuring how the intensity of a laser beam changes after reflecting off a mirror for different angles. If the laser and the detector are 0.5 m away from the mirror and the laser makes an angle of 60° with the mirror, what is the length of the path the laser light travels?

Hint: When light reflects on a plane (flat) mirror, it reflects back at the same angle it came in.

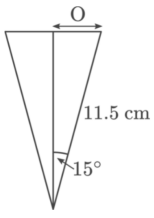


QUESTION 4

Andy is driving his normal route to work on a straight highway when he comes across a road closure. He decides to measure some information about this detour route.
He first travels 200 m at a bearing of 030o. He then slightly changes course and heads on a bearing of 010o for 150 m. Finally, he changes course to a bearing of 330o for another 252 m. This puts him back onto the straight highway he started on.
Calculate how much longer the detour is.

ANSWER - 2 - 1

When viewed from the side in 2D, our cone looks like an isosceles triangle. If we draw a line down the centre of the cone, we split our isosceles triangle into two right angled triangles.



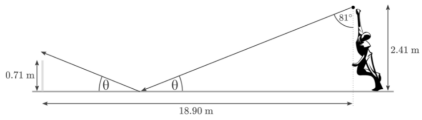
To find the radius of the top of the cone, all we need to do is find the opposite side of one of these triangles, using our new angle of 15°, obtained by halving the original angle.

$$\begin{aligned} \sin(\theta) &= \frac{O}{H} \\ H \times \sin(\theta) &= O \\ O &= H \times \sin(\theta) \\ O &= 11.5 \times \sin(15^\circ) \\ \therefore &= 2.976419\ldots \end{aligned}$$

ANSWER - 4 - 1

ANSWER - 5 - 1

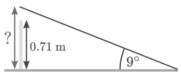
The information we know can be condensed into the following diagram:



We can easily calculate θ using the fact that the angles in a triangle add up to 180°:

$$180^\circ - 90^\circ - 81^\circ = 9^\circ$$

Now that we know the angle at which the ball approaches its bounce, we also know the angle it leaves the bounce (they are equal). Let's add this information to the triangle on the left hand side of the above diagram:



THANK YOU

Thank You :D

QUESTION 2

Find the approximate volume of the scoop of ice cream on top of the cone.

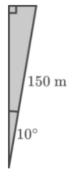
The formula for the volume of a sphere is:

$$V = \frac{4}{3}\pi r^3.$$

QUESTION 5

ANSWER - 4 - 2

Moving on to the second triangle, for the second part of the detour:

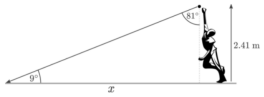


$$\begin{aligned} \cos(\theta) &= \frac{A}{H} \\ H \times \cos(\theta) &= A \\ A &= 150 \times \cos(10^\circ) \\ \therefore &= 147.721163\ldots \end{aligned}$$

ANSWER - 5 - 2

We know the height of the wickets, but we do not know what height the ball will be at when it reaches the wickets. If the height is less than or equal to the height of the wickets, the ball will hit the wickets. If the height is greater than this, the ball will pass over the wickets.

Therefore, we need to solve for the vertical side of this triangle, which will give us the height of the ball as it passes the wickets, thus determining whether they collide or not. However, we still need one more side length to be able to solve for this. We know the overall length of the pitch, so if we can work out the horizontal side of one of the triangles in the first diagram, we can work out both.



The triangle on the bowler's side contains known angles, and a known side length (the adjacent side). Let's calculate the horizontal side of this triangle (let's call it x) using 9° as our angle of reference, making the horizontal side the adjacent side:

$$\begin{aligned} \tan(\theta) &= \frac{O}{A} \\ A &= \frac{O}{\tan(\theta)} \\ x &= \frac{O}{\tan(\theta)} \\ \therefore &= \frac{2.41}{\tan(9^\circ)} \\ \therefore &= 15.216141\ldots \\ \therefore &\approx 15.22\ \text{m} \end{aligned}$$

QUESTION 3

An airplane departs A and flies on a 143° course for 368 km to B. It then changes directions to a 233° course and flies a further 472 km to C.
Find:
1) the distance of C from A
2) the bearing of C from A

ANSWER - 1

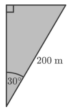
In order to find how much longer the detour route is, we must calculate the length of both routes.

First let's calculate the length of the detour route:

$$\begin{aligned} L_{\text{detour}} &= 200 + 150 + 252 \\ &= 602 \text{ m} \end{aligned}$$

Now, let's calculate the length of the original route using each of the 3 triangles formed by the detour route. For each of these triangles we want to find the vertical side; that is, the one that points in the same direction as the highway route. Adding up the vertical sides from each of the 3 triangles will give us the length of the highway route.

Starting with the first triangle, for the first part of the detour:



$$\begin{aligned} \cos(\theta) &= \frac{A}{H} \\ H \times \cos(\theta) &= A \\ A &= 200 \times \cos(30^\circ) \\ \therefore &= 173.205081\ldots \end{aligned}$$

For P.E. class, Josh is studying his bowling technique in cricket. He finds that he releases the ball just after the apex of his arm rotation, causing the ball to travel at an angle of 81° from the vertical. Josh also measures the height that his arm reaches at this release point, getting a value of 2.41 m. He knows the distance from the bowling crease to the wicket is 18.9 m, and the wickets are 71.1 cm tall. Assume that the ball travels fast enough that the trajectory is a straight line, and that the ball will bounce off the pitch at the same angle it came from. Calculate the height of the ball will bounce to at the wickets and hence determine whether the ball will hit them.

ANSWER - 2 - 2

The radius of the cone will give us the approximate radius of the sphere of ice cream.

Using the volume formula provided, we can calculate the volume of ice cream:

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 2.976419\ldots^3 \\ &= 110.451298\ldots \\ &\approx 110 \text{ cm}^3 \end{aligned}$$

Therefore, we can estimate that the volume of ice cream on the cone is 110 cm^3, rounded to the nearest whole number.

The hyotenuse of this triangle is the path length from the laser to the mirror, so we must calculate that. We know an angle, and the adjacent side, and want to find the hypotenuse, so we must use Cosine.

$$\begin{aligned} \cos(\theta) &= \frac{A}{H} \\ \cos(30^\circ) &= \frac{0.5 \text{ m}}{H} \\ H \times \cos(30^\circ) &= 0.5 \text{ m} \\ H &= \frac{0.5 \text{ m}}{\cos(30^\circ)} \\ \therefore &= 0.577350\ldots \\ \therefore &\approx 0.58 \text{ m} \end{aligned}$$

All we need to do now is multiply this path length (rounded to 2 decimal places) by 2 to get the total path length of the laser.

$$2 \times 0.58 \text{ m} = 1.15 \text{ m}$$

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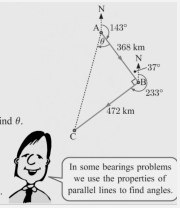
Published 23rd March, 2023.
Last updated 23rd March, 2023.

ANSWER - 3

$\widehat{ABN} = 180° - 143° = 37°$ {co-interior angles}
$\therefore\ \widehat{ABC} = 360° - 37° - 233°$
$= 90°$ {angles at a point}

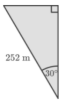
a $AC^2 = 368^2 + 472^2$ {Pythagoras}
$\therefore\ AC = \sqrt{368^2 + 472^2}$ {as $AC > 0$}
≈ 598.5
So, C is about 598.5 km from A.

b To find the bearing of C from A, we first need to find θ.
Now $\tan\theta = \frac{472}{368}$ $\left\{\tan\theta = \frac{\text{OPP}}{\text{ADJ}}\right\}$
$\therefore\ \theta = \tan^{-1}\left(\frac{472}{368}\right)$
$\therefore\ \theta \approx 52.1°$
The bearing of C from A is $143° + 52.1° \approx 195°$.



ANSWER - 4 - 3

For the third and final triangle:



$$\begin{aligned}\cos(\theta) &= \frac{A}{H}\\ H \times \cos(\theta) &= A\\ A &= 252 \times \cos(30°)\\ \therefore &= 218.238402\ldots\end{aligned}$$

Adding up these three distances gives the overall distance of the highway route:

$$\begin{aligned}L_{\text{highway}} &= 173.205081\ldots + 147.721163\ldots + 218.238402\ldots\\ &= 539.164645\ldots\end{aligned}$$

To calculate the extra distance travelled by taking the detour, we simply just take the difference between L_{detour} and L_{highway}:

$$\begin{aligned}L_{\text{detour}} - L_{\text{highway}} &= 602 - 539.164645\ldots\\ &= 62.835355\ldots\\ &\approx 63\text{m}\end{aligned}$$

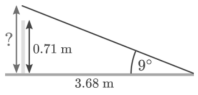
Therefore, the extra distance Andy travelled by taking the detour was 63 m, rounded to the nearest whole number.

ANSWER - 5 - 3

Now, taking the difference between this and the length of the overall pitch will give the horizontal side (we will call it y) of the other triangle:

$$\begin{aligned}y &= 18.9 - 15.22\\ &= 3.68\ \text{m}\end{aligned}$$

Looking back at the wickets triangle:



We can now solve for the vertical side (using $9°$ as the reference angle, so it is also the opposite side):

$$\begin{aligned}\tan(\theta) &= \frac{O}{A}\\ O &= A \times \tan(\theta)\\ \therefore &= 3.68 \times \tan(9)\\ \therefore &= 0.582855\ldots\\ \therefore &\approx 0.58\ \text{m}\end{aligned}$$

This is less than wicket height of 0.71 m, therefore, the ball hits the wicket.