4.1 Definitions/Things that are clear

A ***Polynomial with Coefficients*** in R (Let R be any ring)	An expression of the form: $a0+a1x+a2x^2+...+a_n x^n$ where ***n*** is a **nonnegative integer** and $a_i \in R$
Expression: $a0 + a1x + a_2 x^2 +...+ a_n x^n$	An expression of this form makes sense, provided that the a_i and ***x*** are all elements of some larger ring
In Thm 4.1, **the elements of the ring P** are called	*polynomials with coefficients in R*
In Thm 4.1, **the elements a_i** are called	*coefficients*
In Thm 4.1, **the special element *x*** is called an	*indeterminate*

4.1 Theorems & Corollaries

Theorem 4.1

If R is a ring, then there exists a ring P that contains an element x that is not in R and has these properties:

(i) *R is a subring of P.*
(ii) $xa = ax$ *for every* $a \in R$
(iii) *Every element of P can be written in the form* $a0+a1x+a2x^2+...+a_n x^n$ *for some* $n \geq 0$ *and* $a_i \in R$
(iv) *The representation of elements in P in* ***(iii)*** *is unique in this sense: if* $n \leq m$ *and* $a0+a1x+a2x^2+...+a_n x^n = b0+b1x+b2x^2+...+b_m x^m$, *then* $a_i = b_i$ *for* $i \leq n$ *and* $b_i = 0R$ *for each* $i > n$.
(v) $a0+a1x+a2x^2+...+a_n x^n = 0R$ *if and only if* $a_i = 0R$ *for every* ***i***.

Theorem 4.2