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Linear Algebra Cheat Sheet by gustavhelms via cheatography.com/146840/cs/31828/

Linear Systems		
Solution	Can have no solution , one solution or infinitly many . The solution is the inters - erction	
Solution set	The set of all possible solutions	
Consistency	A system is consistent if there is at least one solution otherwise it is inconsistent	
Equivalent	Linear systems are equivalent if they have the same solution set	
Row operations	Replacement, interchange and scaling	
Row equivalent	If there is a sequence of row operations between two linear systems then the systems are row equivalent. Systems that are row equivalent has the same solution set.	
Existence	If a system has a solution (i.e. consistent)	
Uniqueness	Is the solution unique	
Homogenous	A system is homogenous if it can be written in the form $A\mathbf{x} = 0$	
Trivial solution	If a system only has a the solution $\mathbf{x} = 0$. A system with no free variable only have the trivial solution.	
Non-trivial solution	A nonzero vector that satisfies A x = 0. Has free variable.	
Inverser of a Matrix		
C is invertible if CA = I^{n} and AC = I^{n}		

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C is invertible if CA = 1° and AC = 1°

If A is (2x2) then, A^{-1} =

(A^{-1})^{-1} = A

(AB)^{-1} = B^{-1}A^{-1}

(A^{T})^{-1} = (A^{-1})^{T}
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Tranfo- rmatio- n/m- apping	$T(x)$ from R^n to R^m
Image	For x in \mathbb{R}^{n} the vector T(x) in \mathbb{R}^{m} is called the image
Range	The set of all images of the vectors in the domain of T(x)
Criterion for a transf- ormation to be linear	1. T(u + v) = T(u) + T(v) 2. T(cU) = cT(U)
Standard Matrix	The matrix A for a linear transformation T, that satisfies $T(x) = Ax$ for all x in R^n
Onto	A mapping T is said to be onto if each b in the codomain is the image of at least one x in the domain. Range = Codomain. Solution existance. ColA must match codomain.
One-to- one	If each b in the codomain is only the image at most one x in the domain . Solution Unique- ness.
	T is one-to-one if and only if the cols of A are linearly indepe- ndent
Free variable?	If the system has a free variable, then the system is not one-to-one. I.e. the homogenous system only has the trivial solution
Pivot in every row?	Then T is onto

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Linear Transformation	s (cont)	
Pivot in every column?	Then T is one-to- one	
To determine whether a vector c is in the range of a T . Solution: Let $T(x) = Ax$. Solve the matrix equation $Ax = c$. If the system is consistent , then c is in the range of T.		
The Invertible Matrix T	heorem	
The following statemen either they are all true (<i>n</i> x <i>n</i>) matrix	nts are equivalent i.e. or all false. Let A be a	
A is an invertible matrix	х.	
A is row equivalent to t matrix	the <i>n</i> × <i>n</i> identity	
A has n pivot positions		
The equation $A\mathbf{x} = 0$ has trivial solution	as only the	
The columns of A form ndent set	a linearly indepe-	
The linear transformation one-to-one.	ion $\mathbf{x} \mapsto A\mathbf{x}$ is	
The equation $A\mathbf{x} = \mathbf{b}$ has solution for each \mathbf{b} in F	as at least one R ⁿ	
The columns of A spar	ו R ⁿ	
The linear transformati maps R ⁿ onto R ⁿ	ion $\mathbf{x} \mapsto A\mathbf{x}$	
There is an $n \times n$ matrix $\Box A = I$.	C such that \Box	
There is an $n \times n$ matrix $\Box D = I$.	D such that \Box	
A^{T} is an invertible matrix	rix.	
The columns of A form	a basis of R^n	
Col A = R^n		
Dim Col A = n		

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Rank A = n

Nul A = $\{0\}$

Yours!

Dim Nul A = 0

The number 0 is not an eigenvalue of A

The determinant of A is not 0

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Elementary Matrices

Elementary Matrix Is obtained by performing a single elementary row operation on an identity matrix

Each elementary matrix E is invertible

A nxn matrix A is invertible if and only if A is row equivalent to Iⁿ.

 $A = E^{-1}I^{n}$ and $A^{-1} = EI^{n} = I^{n}$

Row reduce the augmented matrix [A I] to [I A^{-1}]

NOTE If A is not row equivalent to I then A is not invertible

Linear Independence

A set of vectors are **linearly independent** if they cannot be created by any linear combinations of earlier vectors in the set.

If a set of vectors are **linear independent**, then the solution is **unique**

If the vector equeation $c1v1 + c2v2 + ... + cp^*vp = 0$ only has a **trivial solution** the set of vectors are **linearly independent**

Theorem: If a set contains more vectors than there are entries in each vector, then the set is **linearly dependent**

Theorem: If a set of vectors containt the zero vector, then the set is **linearly** dependent

Algebraic properties of a matrix

Algebraic properties of a matrix (cont)

 $(AB)^{\mathsf{T}} = B^{\mathsf{T}} A^{\mathsf{T}}$

For any scalar r, $(rA)^T = rA^T$

LU Factorization

Factorization of a matrix A is an equation that expresses A as a **product** of two or more matrices:

Synthesis: BC = A Analysis: A= BC

Assumption: A is a *m*x*n* matrix that can be row reduced without **interchanges**

L: is a *m*x*m* unit lower triangular with 1's on the diagonal

U: is a *m*x*n* echelon form of A

U is equal to $E^*A = U$, why $A = E^{-1}U = LU$ where $L = E^{-1}$

See figure ** for how to find L and U

Find **x** by first solving Ly = b and then solving Ux = y

Row Reduction and Echelon forms

Leading entry	A leading entry refers to the leftmost non-zero entry in a row
Echelon form	Row equivalent systems can be reduced into several different echelon forms
Reduced echelon form	A system is only row equivalent to one REF
Forward phase	Reducing an augmented matrix A into an echelon form
Backward phase	Reducing an augmented matrix A into a reduced echelon form
Basic variables	Variables in pivot columns .
Free variables	Variables that are not in pivot columns . When a system has a free variable the system is consistent but not unique

Subspaces of Rⁿ

A **subspace** of Rⁿ is any set H in Rⁿ that has three properties:

- The zero vector is in H

- For each ${\bf u}$ and ${\bf v}$ in H, the sum ${\bf u}$ + ${\bf v}$ is in H

- For each **u** in *H* and each scalar *c*, the vector *c***u** is in *H*

Zero subspace is the set containing only the **zero vector** in \mathbb{R}^n

Column space is the set of all linear combinations of the columns of A.

Null space (Nul A) is the set of all solutions of the equation Ax = 0

Basis for a subspace H is the set of linearly independent vectors that span H

In general, the **pivot columns** of A form a basis for col A

The number of vectors in any basis is **unique**. We call this number **dimension**

The **rank** of a matrix *A*, denoted by **rank** *A*, is the **dimension** of the **column space** of *A*

Determine whether b is in the col A. Solution: b is only in col A if the equation Ax = b has a solution

Matrix and vector sum	$A(\mathbf{u} + \mathbf{v}) = A\mathbf{u} + A\mathbf{v}$
Matrix, vector and scalar	$A(c\mathbf{u}) = c(A\mathbf{u})$
Associ- ative law	A(BC) = (AB)C
Left distri- butive law	A (B + C) = AB + AC
Right distributive law	(B + C) = BA + BC
Scalar multiplic- ation	r(AB) = (rA)B = A(rB)
ldentity matrix multi	$I^m A = A = A I^n$
Commute	If AB = BA then we say that A and B commute with each others
	$(A^T)^T = A$
	$(A + B)^{T} = A^{T} + B^{T}$



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Algebraic properties of a vector

u + v = v + u (u + v) + w = u + (v + w) u + (-u) = -u + u c(u + v) = cu + cvc(du) = (cd)u



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