| Linear Systems |  |
| :--- | :--- |
| Solution | Can have no solution, one <br> solution or infinitly many. <br> The solution is the inters- <br> erction |
| Solution set | The set of all possible <br> solutions |
| Consistency | A system is consistent if <br> there is at least one solution <br> otherwise it is inconsistent |
| Equivalent | Linear systems are <br> equivalent if they have the <br> same solution set |
| Row | Replacement, interchange <br> and scaling |
| operations |  |
| Row there is a sequence of |  |
| equivalent | If <br> row operations between <br> two linear systems then the <br> systems are row equivalent. <br> Systems that are row <br> equivalent has the same <br> solution set. <br> If a system has a solution <br> (i.e. consistent) |
| Is the solution unique |  |

## Inverser of a Matrix

C is invertible if $C A=1^{n}$ and $A C=1^{n}$
If $\mathbf{A}$ is $(2 \times 2)$ then, $\mathrm{A}^{-1}=$
$\left(A^{-1}\right)^{-1}=A$
$(A B)^{-1}=B^{-1} A^{-1}$
$\left(A^{\top}\right)^{-1}=\left(A^{-1}\right)^{\top}$


By gustavhelms

| Linear Transformations |  |
| :---: | :---: |
| Tranfo- <br> rmatio- <br> n/m- <br> apping | $T(x)$ from $R^{n}$ to $R^{m}$ |
| Image | For x in $\mathrm{R}^{\mathrm{n}}$ the vector $\mathrm{T}(\mathrm{x})$ in $R^{m}$ is called the image |
| Range | The set of all images of the vectors in the domain of $\mathrm{T}(\mathrm{x})$ |
| Criterion for a transformation to be linear | 1. $T(u+v)=T(u)+T(v)$ <br> 2. $T(c U)=c T(U)$ |
| Standard <br> Matrix | The matrix A for a linear transformation $T$, that satisfies $T(x)=$ Ax for all $x$ in $R^{n}$ |
| Onto | A mapping T is said to be onto if each $b$ in the codomain is the image of at least one x in the domain. Range = Codomain. Solution existance. ColA must match codomain. |
| One-toone | If each $b$ in the codomain is only the image at most one $x$ in the domain. Solution Uniqueness. |
|  | $T$ is one-to-one if and only if the cols of $A$ are linearly independent |
| Free variable? | If the system has a free variable, then the system is not one-to-one. I.e. the homogenous system only has the trivial solution |
| Pivot in every row? | Then T is onto |

Linear Transformations (cont)

| Pivot in every | Then T is one-to- |
| :--- | :--- |
| column? | one |

To determine whether a vector $\mathbf{c}$ is in the range of a $T$. Solution: Let $T(x)=A x$. Solve the matrix equation $A x=c$. If the system is consistent, then c is in the range of T .

## The Invertible Matrix Theorem

The following statements are equivalent i.e. either they are all true or all false. Let A be a ( $n \times n$ ) matrix
$A$ is an invertible matrix.
$A$ is row equivalent to the $n \times n$ identity matrix
$A$ has $n$ pivot positions.
The equation $A \mathbf{x}=\mathbf{0}$ has only the trivial solution

The columns of $A$ form a linearly independent set

The linear transformation $\mathbf{x} \mapsto A \mathbf{x}$ is one-to-one.

The equation $A \mathbf{x}=\mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathrm{R}^{\mathrm{n}}$

The columns of $A$ span $\mathrm{R} \quad \mathrm{n}$
The linear transformation $\mathbf{x} \mapsto A \mathbf{x}$
maps $R^{n}$ onto $R^{n}$
There is an $n \times n$ matrix $C$ such that $\square A=I$.

There is an $n \times n$ matrix $D$ such that $\square D=I$.
$A^{\top}$ is an invertible matrix.
The columns of $A$ form a basis of $R^{\wedge} n$
$\operatorname{Col} A=R \wedge n$
$\operatorname{Dim} \operatorname{Col} A=n$
Rank $\mathrm{A}=\mathrm{n}$
Nul $A=\{0\}$
Dim Nul A $=0$
The number 0 is not an eigenvalue of $A$
The determinant of A is not 0

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## Elementary Matrices

Elementary Matrix Is obtained by performing a single elementary row operation on an identity matrix
Each elementary matrix $E$ is invertible
A nxn matrix $A$ is invertible if and only if $A$ is row equivalent to $I^{n}$.
$A=E^{-1} I^{n}$ and $A^{-1}=E I^{n}=I^{n}$
Row reduce the augmented matrix [ A I ] to [ $\|^{-1}$ ]
NOTE If A is not row equivalent to I then A is not invertible

## Linear Independence

A set of vectors are linearly independent if they cannot be created by any linear combinations of earlier vectors in the set.

If a set of vectors are linear independent, then the solution is unique If the vector equeation c1v1 $+c \mathbf{v} \mathbf{v} 2+\ldots+$ $c p^{*} v p=0$ only has a trivial solution the set of vectors are linearly independent

Theorem: If a set contains more vectors than there are entries in each vector, then the set is linearly dependent
Theorem: If a set of vectors containt the zero vector, then the set is linearly dependent

Algebraic properties of a matrix

## Algebraic properties of a matrix (cont)

$(A B)^{\top}=B^{\top} A^{\top}$
For any scalar $r,(r A)^{\top}=r A^{\top}$

## LU Factorization

Factorization of a matrix A is an equation that expresses $A$ as a product of two or more matrices:
Synthesis: BC = A
Analysis: $A=B C$
Assumption: A is a $m \times n$ matrix that can be row reduced without interchanges
L : is a $m \times m$ unit lower triangular with 1 's on the diagonal

U : is a $m \times n$ echelon form of A
U is equal to $\mathrm{E}^{*} \mathrm{~A}=\mathrm{U}$, why $\mathrm{A}=\mathrm{E}^{1} \mathrm{U}=\mathrm{LU}$ where $\mathrm{L}=\mathrm{E}^{-1}$

See figure ** for how to find $L$ and $U$
Find $\mathbf{x}$ by first solving $\mathbf{L y}=\mathbf{b}$ and then solving Ux =y

| Row Reduction and Echelon forms |  |
| :--- | :--- |
| Leading <br> entry | A leading entry refers to the <br> leftmost non-zero entry in a <br> row |
| Echelon | Row equivalent systems can <br> be reduced into several <br> fifferent echelon forms |
| Reduced | A system is only row |
| echelon | equivalent to one REF |
| form |  |$\quad$| Forward | Reducing an augmented matrix |
| :--- | :--- |
| phase | A into an echelon form |

## Subspaces of R^n

A subspace of $R^{n}$ is any set $H$ in $R^{n}$ that has three properties:

- The zero vector is in H
- For each $\mathbf{u}$ and $\mathbf{v}$ in H , the sum $\mathbf{u}+\mathbf{v}$ is in H
- For each $\mathbf{u}$ in $H$ and each scalar $c$, the vector Cu is in $H$

Zero subspace is the set containing only the zero vector in $\mathrm{R}^{\mathrm{n}}$

Column space is the set of all linear combinations of the columns of A .

Null space (Nul A) is the set of all solutions of the equation $\mathbf{A x}=\mathbf{0}$

Basis for a subspace H is the set of linearly independent vectors that span H

In general, the pivot columns of A form a basis for col A

The number of vectors in any basis is unique. We call this numberdimension

The rank of a matrix $A$, denoted by rank
$A$, is the dimension of the column space of $A$

Determine whether $b$ is in the col $A$.
Solution: $b$ is only in col $A$ if the equation $A x$
$=\mathrm{b}$ has a solution

Matrix and $A(u+v)=A u+A v$
vector
sum
Matrix, $\quad \mathrm{A}(c \mathbf{u})=c(\mathrm{Au})$
vector and
scalar
Associ- $\quad A(B C)=(A B) C$
ative law
Left distri- $\quad A(B+C)=A B+A C$
butive law
Right $\quad(B+C)=B A+B C$
distributive
law
Scalar $\quad r(A B)=(r A) B=A(r B)$
multiplic-
ation
Identity $\quad \|^{m} A=A=A P$
matrix
multi
Commute If $A B=B A$ then we say that $A$ and $B$ commute with each others
$\left(A^{\top}\right)^{\top}=A$
$(A+B)^{\top}=A^{\top}+B^{\top}$

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Algebraic properties of a vector
$u+v=v+u$
$(u+v)+w=u+(v+w)$
$\mathbf{u}+(\mathbf{u})=-\mathbf{u}+\mathbf{u}$
$c(\mathbf{u}+\mathbf{v})=c \mathbf{u}+c \mathbf{v}$
$c(d \mathbf{u})=(c d) \mathbf{u}$

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